

Hypothesis Tests Requiring Unbiased Estimators and Central Limit Theorem (From OCR 4733)

Q1, (Jan 2006, Q6)

Alex obtained the actual waist measurements, w inches, of a random sample of 50 pairs of jeans, each of which was labelled as having a 32-inch waist. The results are summarised by

$$n = 50, \quad \Sigma w = 1615.0, \quad \Sigma w^2 = 52\,214.50.$$

Test, at the 0.1% significance level, whether this sample provides evidence that the mean waist measurement of jeans labelled as having 32-inch waists is in fact greater than 32 inches. State your hypotheses clearly. [10]

Q2, (Jun 2008, Q3)

In a factory the time, T minutes, taken by an employee to make a single item is a normally distributed random variable with mean 28.0. A new ventilation system is installed, after which the times taken to produce a random sample of 40 items are measured. The sample mean is 26.44 minutes and it is given that $\frac{\Sigma t^2}{40} - 26.44^2 = 37.05$. Test, at the 10% significance level, whether there is evidence of a change in the mean time taken to make an item. [8]

Q3, (Jan 2009, Q7)

A motorist records the time taken, T minutes, to drive a particular stretch of road on each of 64 occasions. Her results are summarised by

$$\Sigma t = 876.8, \quad \Sigma t^2 = 12\,657.28.$$

- (i) Test, at the 5% significance level, whether the mean time for the motorist to drive the stretch of road is greater than 13.1 minutes. [11]
- (ii) Explain whether it is necessary to use the Central Limit Theorem in your test. [1]

Q4, (Jan 2011, Q4)

The continuous random variable X has mean μ and standard deviation 45. A significance test is to be carried out of the null hypothesis $H_0: \mu = 230$ against the alternative hypothesis $H_1: \mu \neq 230$, at the 1% significance level. A random sample of size 50 is obtained, and the sample mean is found to be 213.4.

- (i) Carry out the test. [5]
- (ii) Explain whether it is necessary to use the Central Limit Theorem in your test. [2]

Q5, (Jun 2011, Q6)

Records show that before the year 1990 the maximum daily temperature $T^\circ\text{C}$ at a seaside resort in August can be modelled by a distribution with mean 24.3. The maximum temperatures of a random sample of 50 August days since 1990 can be summarised by

$$n = 50, \quad \Sigma t = 1314.0, \quad \Sigma t^2 = 36\,602.17.$$

- (i) Test, at the 1% significance level, whether there is evidence of a change in the mean maximum daily temperature in August since 1990. [11]
- (ii) Give a reason why it is possible to use the Central Limit Theorem in your test. [1]

Q6, (Jun 2012, Q5)

The acidity A (measured in pH) of soil of a particular type has a normal distribution. The pH values of a random sample of 80 soil samples from a certain region can be summarised as

$$\Sigma a = 496, \quad \Sigma a^2 = 3126.$$

Test, at the 10% significance level, whether in this region the mean pH of soil is 6.1.

[11]

Q7, (Jun 2014, Q7)

An examination board is developing a new syllabus and wants to know if the question papers are the right length. A random sample of 50 candidates was given a pre-test on a dummy paper. The times, t minutes, taken by these candidates to complete the paper can be summarised by

$$n = 50, \quad \Sigma t = 4050, \quad \Sigma t^2 = 329\,800.$$

Assume that times are normally distributed.

- (i) Estimate the proportion of candidates that could not complete the paper within 90 minutes. **[6]**
- (ii) Test, at the 10% significance level, whether the mean time for all candidates to complete this paper is 80 minutes. Use a two-tail test. **[7]**
- (iii) Explain whether the assumption that times are normally distributed is necessary in answering
 - (a) part (i),
 - (b) part (ii). **[2]**

Q8, (Jun 2015, Q6)

Records for a doctors' surgery over a long period suggest that the time taken for a consultation, T minutes, has a mean of 11.0. Following the introduction of new regulations, a doctor believes that the average time has changed. She finds that, with new regulations, the consultation times for a random sample of 120 patients can be summarised as

$$n = 120, \Sigma t = 1411.20, \Sigma t^2 = 18\,737.712.$$

- (i) Test, at the 10% significance level, whether the doctor's belief is correct. **[11]**
- (ii) Explain whether, in answering part (i), it was necessary to assume that the consultation times were normally distributed. **[1]**

Q9, (Jun 2016, Q8)

It is known that the lifetime of a certain species of animal in the wild has mean 13.3 years. A zoologist reads a study of 50 randomly chosen animals of this species that have been kept in zoos. According to the study, for these 50 animals the sample mean lifetime is 12.48 years and the population variance is 12.25 years².

- (i) Test at the 5% significance level whether these results provide evidence that animals of this species that have been kept in zoos have a shorter expected lifetime than those in the wild. **[7]**
- (ii) Subsequently the zoologist discovered that there had been a mistake in the study. The quoted variance of 12.25 years² was in fact the sample variance. Determine whether this makes a difference to the conclusion of the test. **[5]**
- (iii) Explain whether the Central Limit Theorem is needed in these tests. **[1]**