Continuous Uniform Distribution (Fom Edexcel 6684)

Q1, (Jun 2005, Q2)

(a)	$f(x) = \frac{1}{4}$, $2 \le x \le 6$	$\frac{1}{4}$ and range	B1	
	$= 0$, otherwise	0 and range	B1	
				(2)
(b)	E(X) = 4 by symmetry or formula	4	B1	
				(1)
(c)	$\text{Var}(X) = \frac{(6-2)^2}{12}$	Use of formula	M1	
	$= \frac{4}{3}$	$1.\dot{3}$ or $1\frac{1}{3}$ or $\frac{4}{3}$ or 1.33	A1	
				(2)
(d)	$F(x) = \int_2^x \frac{1}{4} dt = \left[\frac{1}{4}t\right]_2^x$	Use of $\int f(x)\,dx$	M1	
	$= \frac{1}{4}(x-2)$	$\frac{1}{4}(x-2)$ or equiv.	A1	
	$F(x) = \frac{1}{4}(x-2)$, $2 \le x \le 6$	$\frac{1}{4}(x-2)$ and range	B1ft	
	$= 1$, $x > 6$ $= 0$, $x < 2$	ends and ranges	B1	
				(4)
(e)	$P(2.3 < X < 3.4) = \frac{1}{4}(3.4 - 2.3)$	Use of area or F(x)	M1	
	$= 0.275$	0.275 or $\frac{11}{40}$	A1	(2)
				Total 11

Q2, (Jun 2006, Q2)

(a)	$P(L < -2.6) = 1.4 \times \frac{1}{8} = \frac{7}{40}$ or 0.175 or equivalent		B1	
				(1)
(b)	$P(L < -3.0 \text{ or } L > 3.0) = 2 \times \left(1 \times \frac{1}{8}\right) = \frac{1}{4}$	M1 for 1/8 seen	M1;A1	
				(2)
(c)	P (within 3mm) $= 1 - \frac{1}{4} = 0.75$ B(20,0.75)	recognises binomial	B1	
		Using B(20,p)	M1	
	Let X represent number of rods within 3mm			
	$P(X \le 9 / p = 0.25)$ or $1 - P(X \le 10 / p = 0.75)$		M1	
	$= 0.9861$	awrt 0.9861	A1	
				(4)

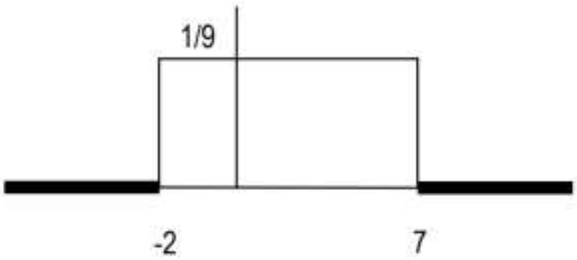
Q4, (Jan 2007, Q5)

(a)	$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha}, & \alpha < x < \beta, \\ 0, & \text{otherwise.} \end{cases}$	function including inequality, 0 otherwise	B1,B1 (2)
(b)	$\dfrac{\alpha+\beta}{2} = 2, \quad \dfrac{3-\alpha}{\beta-\alpha} = \dfrac{5}{8}$	or equivalent	B1,B1
	$\alpha + \beta = 4$ $3\alpha + 5\beta = 24$		
	$3(4-\beta) + 5\beta = 24$ $2\beta = 12$ $\beta = 6$	attempt to solve 2 eqns	M1
	$\alpha = -2$	both	A1 (4)
(c)	$E(X) = \dfrac{150+0}{2} = 75\text{ cm}$	75	B1 (1)
(d)	Standard deviation = $\sqrt{\dfrac{1}{12}(150-0)^2}$		M1
	$= 43.30127\ldots\text{cm}$	$25\sqrt{3}$ or awrt 43.3	A1 (2)
(e)	$P(X < 30) + P(X > 120) = \dfrac{30}{150} + \dfrac{30}{150}$	1st or at least one fraction, + or double	M1,M1
	$= \dfrac{60}{150}$ or $\dfrac{2}{5}$ or 0.4 or equivalent fraction		A1 (3)
			Total 12

Q5, (Jan 2009, Q2)

(a) $f(x)=\begin{cases}\frac{1}{9} & -2\le x\le 7\\ 0 & otherwise\end{cases}$ B1 B1 (2)

(b)



B1 B1 (2)

(c) $E(X)=\underline{2.5}$ Var $(X)=\frac{1}{12}(7+2)^2$ or $\underline{6.75}$ both B1

$E(X^2)=\text{Var}(X)+E(X)^2$ M1

$=6.75+2.5^2$

$=13$ A1 (3)

alternative

$\int_{-2}^{7} x^2 f(x)dx=\left[\frac{x^3}{27}\right]_{-2}^{7}$ "$\int x^2 f(x)$" attempt to integrate and use limits of -2 and 7 B1 M1

$=13$ A1

(d) $P(-0.2<X<0.6)=\frac{1}{9}\times 0.8$ M1

$=\frac{4}{45}$ or 0.0889 0r equiv awrt 0.089 A1 (2)

Q6, (Jun 2010, Q3)

Method 1	Method 2	Method 3	
$P(X > 6) = \frac{1}{6}$	$P(4 < X < 6) = \frac{1}{3}$	$P(X > 6) = \frac{1}{6}$	B1 M1
$P(X < 4) = \frac{1}{2}$		$Y \sim U[3,9]$ $P(Y > 6) = \frac{1}{2}$	A1
$\text{total} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	$1 - \frac{1}{3} = \frac{2}{3}$	$\text{total} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$	M1dep B A1
			(5)
			[5]

Notes

Methods 1 and 2

B1 for 6 and 4 (allow if seen on a diagram on x-axis)

M1 for $P(X > 6)$ or $P(6 < X < 7)$; or $P(X < 4)$ or $P(1 < X < 4)$; or $P(4 < X < 6)$

Allow $\leq$ *and* $\geq$ signs

A1 $\frac{1}{6}$; *or* $\frac{1}{2}$; $\frac{1}{3}$ must match the probability statement

M1 for adding their " $P(X > 6)$" and their "$P(X < 4)$" or 1 - their "$P(4 < X < 6)$" dep on getting first B mark

A1 cao $\frac{2}{3}$

Method 3 $Y \sim U[3, 9]$

B1 for 6 with U[1,7]and 6 with U[3,9]

M1 for $P(X > 6)$ or $P(6 < X < 7)$ or $P(6 < Y < 9)$

A1 $\frac{1}{6}$; *or* $\frac{1}{2}$; must match the probability statement

M1 for adding their " $P(X > 6)$" and their "$P(Y > 6)$" dep on getting first B mark

A1 cao $\frac{2}{3}$

Q7, (Jun 2011, Q4)

(a)	$\frac{9.5-7}{10-7}$	M1
	$=\frac{5}{6}$ awrt 0.833	A1
		(2)
(b)	P(Longest > 9.5) = 1 - P(all < 9.5) = $1-\left(\frac{5}{6}\right)^3$	M1
	$=\frac{91}{216}$ or 0.421	A1
		(2)
(c)	P(a stick < 7.6) = $\frac{0.6}{3}=0.2$	B1
	Let Y = number of sticks (out of 6) <7.6 then Y~B(6, 0.2)	M1
	$P(Y>4) = 1 - P(Y \leq 4)$	M1
	= 1 - 0.9984	
	= 0.0016 or $\frac{1}{625}$	A1
		(4)
		8

Notes:

(a) M1 for an expression for the probability e.g. $\int_7^{9.5} \frac{1}{3}\, dx$

(b) M1 for $1-(a)^3$ **or** $(1-a)^3+3(1-a)^2a+3(1-a)a^2$

A1 awrt 0.421

(c) B1 0.2 may be implied by at least one correct probability

1st M1 for writing or using B(6, p) may be implied by $np^x(1-p)^{6-x}$ using their p and $n \geq 1$

2nd M1 for writing or using $1 - P(Y \leq 4)$ or $np^5(1-p)+p^6$ (n is an integer > 1)

A1 cao

NB 0.0016 with no working gets B0M0M0A0

Q8, (Jun 2013(R), Q3)

(a)	$\frac{1}{2}(a+b)=23$ and $\frac{1}{12}(b-a)^2=75$	B1B1	
	$a+b=46$ and $b-a=\sqrt{12\times 75}\,(=30)$	M1	
	Adding gives $2b=76$	M1	
	$\underline{\mathbf{b=38}}$ and $\underline{\mathbf{a=8}}$	A1 A1	
			(6)
	alternative		
	$\frac{1}{2}(a+b)=23$ and $\frac{1}{12}(b-a)^2=75$	B1B1	
	$a+b=46$ and hence $(46-2a)^2=900$ oe	M1	
	$a^2-46a+304=0$		
	$(a-8)(a-38)=0$	M1	
	$\underline{\mathbf{b=38}}$ and $\underline{\mathbf{a=8}}$	A1 A1	
			(6)
(b)	$P(23<X<c)=0.5-0.32$ or $c=28.4$ and prob $=\frac{5.4}{30}$	M1	
	$=\underline{\mathbf{0.18}}$	A1	
			(2)
			[8]

	Notes
(a)	1st B1 for at least one correct equation using given formulae
	2nd B1 for any 2 correct equations for a and b using both 23 and 75
	1st M1 for rearranging to get two linear equations in a and b *or* rearranging and substituting linear equation into quadratic.
	2nd M1 for solving i.e. eliminating one variable leading to a linear equation in one variable *or* solving their quadratic correctly by any method.
	1st A1 for $b=38$
	2nd A1 for $a=8$
	SC If they get $b=8$ and $a=38$ or they give two sets of values and do not eliminate one then they can get B1B1M1M1A1A0
(b)	M1 for a correct method, e.g. a correct expression or seeing calculation for c and calculation for probability
	A1 for 0.18 only

Q8, (Jan 2013, Q4)

(a)	Mean = 1	B1	(1)
(b)	$P(X \le 2.4) = (2.4 - -4) \times \frac{1}{10}$	M1	
	$= 0.64$ or $\frac{16}{25}$	A1	
			(2)
(c)	$P(-3 < X - 5 < 3) = P(2 < X < 6)$	M1	
	$= 0.4$	A1	
			(2)
(d)	$\int_a^{4a} \frac{y^2}{4a - a} dy = \left[\frac{y^3}{9a}\right]_a^{4a}$	M1 M1 dep	
	$= \frac{64a^3 - a^3}{9a}$	A1	
	$= 7a^2$ *AG	A1cso	
			(4)
(e)	$\text{Var}(Y) = \frac{1}{12}(4a - a)^2$ or $\text{Var}(Y) = 7a^2 - \left(\frac{5}{2}a\right)^2$	M1	
	$= \frac{3}{4}a^2$	A1cso	
			(2)
(f)	$\frac{2}{3} = \frac{1}{3a}\left(\frac{8}{3} - a\right)$	M1 A1	
	$a = \frac{8}{9}$	A1	
			(3)
			Total 14

Q9, (Jun 2013, Q4)

(a)	$E(X) = \frac{5b}{2}$	B1	(1)
(b)	$\text{Var}(X) = E(X^2) - (E(X))^2$		
	$= \int_b^{4b} \frac{x^2}{3b} dx - (\frac{5b}{2})^2$	M1	
	$= \left[\frac{x^3}{9b}\right]_b^{4b} - \frac{25b^2}{4}$	M1d	
	$= \frac{63b^3}{9b} - \frac{25b^2}{4}$		
	$= \frac{3b^2}{4}$	A1cso	(3)
(c)	$\text{Var}(3 - 2X) = 4\text{Var}(X)$	M1	
	$= 3b^2$	A1	(2)
(d)	$F(x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{3} & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$	B1B1	(2)
(e)	$\frac{x-1}{3} = 0.5$ so $x = 2.5$	B1	(1)
			Total 9 marks