Continuous Random Variables (Functions of CRVs) (From OCR 4734)

Q1, (Jan 2007, Q6)

The lifetime of a particular machine, in months, can be modelled by the random variable T with probability density function given by

$$f(t) = \begin{cases} \frac{3}{t^4} & t \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Obtain the (cumulative) distribution function of T. [2]

(ii) Show that the probability density function of the random variable Y, where $Y = T^3$, is given by $g(y) = \frac{1}{y^2}$, for $y \geqslant 1$. [6]

(iii) Find $E(\sqrt{Y})$. [3]

Q2, (Jun 2007, Q7)

The continuous random variable X has (cumulative) distribution function given by

$$F(x) = \begin{cases} 0 & x < 1, \\ 1 - \frac{1}{x^4} & x \geqslant 1. \end{cases}$$

(i) Find the (cumulative) distribution function, $G(y)$, of the random variable Y, where $Y = \frac{1}{X^2}$. [4]

(ii) Hence show that the probability density function of Y is given by

$$g(y) = \begin{cases} 2y & 0 < y \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$ [2]

(iii) Find $E(\sqrt[3]{Y})$. [3]

Q3, (Jan 2009, Q4)

The weekly sales of petrol, X thousand litres, at a garage may be modelled by a continuous random variable with probability density function given by

$$f(x) = \begin{cases} c & 25 \leqslant x \leqslant 45, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant. The weekly profit, in £, is given by $(400\sqrt{X} - 240)$.

(i) Obtain the value of c. **[1]**

(ii) Find the expected weekly profit. **[3]**

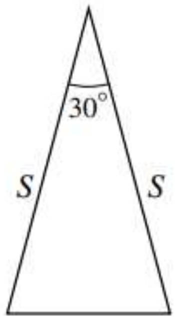
(iii) Find the probability that the weekly profit exceeds £2000. **[3]**

Q4, (Jun 2010, Q8)

The continuous random variable S has probability density function given by

$$f(s) = \begin{cases} \dfrac{8}{3s^3} & 1 \leqslant s \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

An isosceles triangle has equal sides of length S, and the angle between them is 30° (see diagram).



(i) Find the (cumulative) distribution function of the area X of the triangle, and hence show that the probability density function of X is $\dfrac{1}{3x^2}$ over an interval to be stated. **[7]**

(ii) Find the median value of X. **[3]**

Q5, (Jun 2011, Q5)

The continuous random variable X has (cumulative) distribution function given by

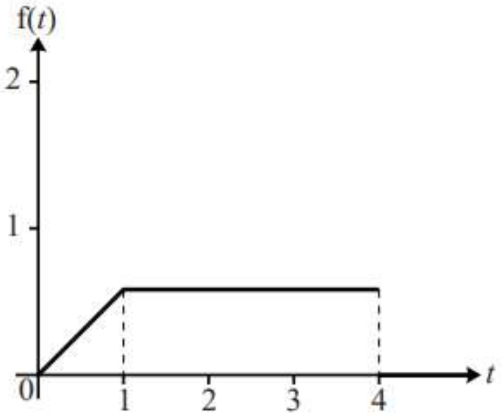
$$F(x) = \begin{cases} 0 & x < 1, \\ \frac{4}{3}\left(1 - \dfrac{1}{x^2}\right) & 1 \leqslant x \leqslant 2, \\ 1 & x > 2. \end{cases}$$

(i) Find the median value of X. **[2]**

(ii) Find the (cumulative) distribution function of Y, where $Y = \dfrac{1}{X^2}$, and hence find the probability density function of Y. **[6]**

(iii) Evaluate $E\left(2 - \dfrac{2}{X^2}\right)$. **[3]**

Q6. (Jun 2012, Q6)



The diagram shows the probability density function f of the continuous random variable T, given by

$$f(t) = \begin{cases} at & 0 \leqslant t \leqslant 1, \\ a & 1 < t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Find the value of a. **[2]**

(ii) Obtain the cumulative distribution function of T. **[4]**

(iii) Find the cumulative distribution of Y, where $Y = T^{\frac{1}{2}}$, and hence find the probability density function of Y. **[7]**

Q7, (Jan 2013, Q4)

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{2}\sqrt{x} & 0 < x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is given by $Y = \frac{1}{\sqrt{X}}$.

(i) Find the (cumulative) distribution function of Y, and hence show that its probability density function is given by

$$g(y) = \frac{3}{y^4},$$

for a set of values of y to be stated. **[7]**

(ii) Find the value of $E(Y^2)$. **[2]**

Q8, (Jun 2013, Q5)

The continuous random variable Y has probability density function given by

$$f(y) = \begin{cases} \ln(y) & 1 \leqslant y \leqslant e, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Verify that this is a valid probability density function. **[4]**

(ii) Show that the (cumulative) distribution function of Y is given by

$$F(y) = \begin{cases} 0 & y < 1, \\ y\ln y - y + 1 & 1 \leqslant y \leqslant e, \\ 1 & \text{otherwise.} \end{cases}$$ **[3]**

(iii) Verify that the upper quartile of Y lies in the interval [2.45, 2.46]. **[2]**

(iv) Find the (cumulative) distribution function of X where $X = \ln Y$. **[4]**

Q9, (Jun 2014, Q9)

A rectangle of area A m^2 has a perimeter of 20 m and each of the two shorter sides are of length X m, where X is uniformly distributed between 0 and 2.

(i) Write down an expression for A in terms of X, and hence show that $A = 25 - (X - 5)^2$. **[3]**

(ii) Write down the probability density function of X. **[1]**

(iii) Show that the cumulative distribution function of A is

$$F(a) = \begin{cases} 0 & a < 0, \\ \frac{1}{2}(5 - \sqrt{25 - a}) & 0 \leqslant a \leqslant 16, \\ 1 & a > 16. \end{cases}$$ **[5]**

(iv) Find the probability density function of A. **[2]**

Q10, (Jun 2016, Q8)

The radius, R, of a sphere is a random variable with a continuous uniform distribution between 0 and 10.

(i) Find the cumulative distribution function and probability density function of A, the surface area of the sphere. **[8]**

(ii) Find $P(A \leqslant 200\pi)$. **[2]**
