Continuous Random Variables (Functions of CRVs) (From OCR 4734)

Q1, (Jan 2007, Q6)

(i) $\int_1^t \frac{3}{x^4}\,dx$ M1 Any variable

$F(t) = \begin{cases} 1-\frac{1}{t^3} & t \ge 1, \\ (0 & \text{otherwise.}) \end{cases}$ A1 **2**

(ii) $G(y) = P(Y \le y)$ M1

$= P(T \le y^{1/3})$ A1

$= F(y^{1/3})$ M1

$= 1 - 1/y$ A1 √ ft $F(t)$

$g(y) = G'(y)$ M1

$= 1/y^2,\ y \ge 1$ AG A1 **6**

(iii) EITHER $\int_1^\infty \frac{\sqrt{y}}{y^2}\,dy$ OR $\int_1^\infty \frac{3t^{\frac{3}{2}}}{t^4}\,dt$ M1

$\left[-2y^{-1/2}\right]_1^\infty$ $\left[-2t^{-3/2}\right]_1^\infty$ B1

$= 2$ A1 **3**

Q2, (Jun 2007, Q7)

(i) $G(y) = P(Y \le y)$ M1 May be implied by following line

$= P(X^2 \ge 1/y)$ [or $P(X > 1/\sqrt{y})$] A1 Accept strict inequalities

$= 1 - F(1/\sqrt{y})$ A1

$= \begin{cases} 0 & y \le 0, \\ y^2 & 0 \le y \le 1, \\ (1 & y > 1.) \end{cases}$

A1 **4** Or $F(x)=P(X \le x) = P(Y \ge 1/x^2)$ M1

$=1 - P(Y < 1/x^2)$ A1

$=1-G(y)$;etc A1 A1

(ii) Differentiate their $G(y)$ M1

to obtain $g(y)= 2y$ for $0 < y \le 1$ AG A1 **2** Only from G correctly obtained

---

(iii) $\int_0^1 2y(\sqrt[3]{y}\,dy$ M1 Unsimplified, but with limits

$=[6y^{7/3}/7]$ B1 OR: Find f(x), $\int_1^\infty x^{-2/3} f(x)dx$ M1

$= {}^6/_7$ A1 **3** $=[4x^{-14/3}/(14/3)]$; ${}^6/_7$ B1A1

OR: Find $H(z)$, $Z = Y^{1/3}$

Q3, (Jan 2009, Q4)

(i)	$c = 1/20$	B1 1	
(ii)	$\int_{25}^{45} \frac{400\sqrt{x} - 240}{20} dx$	M1	
	$= \left[\frac{40}{3}x^{3/2} - 12x\right]$	A1	Correct indefinite integral
	$= 2118(£)$	A'1 3	2120 or better than 2118
(iii)	$400\sqrt{X} - 240 > 2000$, $X > 31.36$	M1	Or 31.4
	$P(X > 31.36) = (45 - 31.36)/20$	M1	
	$= 0.682$	A1 3	cao

Q4, (Jun 2010, Q8)

(i)	$X = \frac{1}{4}S^2$	B1	
	$F(s) = \int_1^s \frac{8}{3s^3} ds = \left[-\frac{4}{3s^2}\right]_1^s$	M1	
	$= \frac{4}{3}(1 - 1/s^2)$	A1	Ignore range here
	$G(x) = P(X \le x) = P(S \le 2\sqrt{x})$ $= F(2\sqrt{x})$	M1	SR: B1 for $G(x)=F(2\sqrt{x})$ without justification and with correct result ft F
	$= \frac{4}{3} - \frac{1}{3x}$	A1 ft	
	$g(x) = \begin{cases} \frac{1}{3x^2} & \frac{1}{4} \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$	M1 B1	For $G'(a)$ For range
		7	
(ii)	EITHER: $G(m) = \frac{1}{2}$	M1	ft $G(x)$ in **(i)**
	$\Rightarrow \frac{4}{3} - \frac{1}{3x} = \frac{1}{2}$	A1 ft	CAO
	$\Rightarrow m = \frac{2}{5}$	A1	
	OR: $\int_{1/4}^{m} \frac{1}{3x^2} dx = \frac{1}{2}$	M1	Allow wrong $\frac{1}{4}$
	$\Rightarrow \left[-\frac{1}{3x}\right]_{1/4}^{m} = \frac{1}{2}$	A1	Allow wrong $\frac{1}{4}$
	$\Rightarrow \quad m = \frac{2}{5}$	A1	CAO
		3 [10]	

Q5, (Jun 2011, Q5)

	Solution	Marks		Notes
(i)	Solve $\frac{4}{3}(1-\frac{1}{m^2}) = \frac{1}{2}$	M1		
	Giving $m = \sqrt{\frac{8}{5}}$	A1	2	Or equivalent. 1.26, 1.265, $2\sqrt{10}/5$
(ii)	$G(y) = P(Y \le y)$ or $<$	M1		Or: $x=1/\sqrt{y}$,
	$= P(X \ge 1/\sqrt{y})$	A1		$\lvert dx/dy \rvert = 1/(2y^{3/2})$ B1
	$= 1 - F(1/\sqrt{y})$	M1		$f(x)=8/(3x^3)$; $1 \le x \le 2$ M1A1
	$= 1 - {}^4/_3(1-y)$ or $(4y-1)/3$	A1		$g(y)=f(x)\lvert dx/dy \rvert$ M1
	$1 \le 1/\sqrt{y} \le 2 \Rightarrow \frac{1}{4} \le y \le 1$	B1		$=4/3$ A1
				$\frac{1}{4} \le y \le 1$ B1
	$g(y) = \begin{cases} 4/3 & 1/4 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$	B1√	6	Ft $G(y)$
(iii)	EITHER: $E(2-2Y)$	M1		
	$= 2 - 2\times{}^5/_8$	A1√		√ $g(y)$
	$= {}^3/_4$	A1		CAO AEF
	OR $2 - \int_1^2 16/(3x^5)dx$ OR $\int_1^2 (2-2/x^2)(8/3x^3)dx$	M1		From $2 - \int xF'(x)dx$
	$= 2 + [4/(3x^4)]$ $=[-8/(3x^2) + 4/(3x^4)]$	A1		√ $f(x)$
	$= 3/4$ $=3/4$	A1	3	CAO AEF
			(11)	

Q6. (Jun 2012, Q6)

(i)	Total area from $t = 0$ to 4 is 1 and $\frac{1}{2}a + 3a = 1$ Solve to give $a = 2/7$	M1 A1 **[2]**	Any method	
(ii)	0 $t<0$ $t^2/7$ $0 \le t \le 1$ "1/7"+ $\int_1^t 2dt/7 = 2t/7 - 1/7$ 1 $t>4$	B1 M1 A1 B1 **[4]**	Ft a for B1 and A1 For $t<0$ and $t>4$	
(iii)	$G(Y) = P(Y \le y) = P(T \le y^2)$ $=F(y^2)$ $= \begin{cases} (0 & y<0,) \\ y^4/7 & 0 \le y \le 1, \\ (2y^2-1)/7 & 1 < y \le 2, \\ (1 & y \ge 2) \end{cases}$ $g(y) = G'(y) = \begin{cases} 4y^3/7 & 0 \le y \le 1 \\ 4y/7 & 1 < y \le 2 \end{cases}$ (0 otherwise)	M1 A1 M1 A1 ft M1 A1 B1 **[7]**	Allow < Seen or implied Possible to score M0A0,then M1A1M1A1B1 , For using F correctly For correct expressions ft a and F(t) For differentiating Correctly,allow from eg $2y^2/7$ Correct ranges for y seen	$\int_0^1 2tdt/7 + \int_1^4 2dt/7 = 1$ M1 $2ydy=dt$ oe M1 $(\int 4y^3dy/7 + \int 4ydy/7 = 1)$ $g(y)=4y^3/7, 4y/7$ A1 $0 \le y \le 1,\ 1 < y \le 2$ B1 $G(y)=y^4/7$ B1 $2y^2/7+c$ and $G(1)=1/7$ M1 $(2y^2-1)/7$ A1

Q7, (Jan 2013, Q4)

(i)	$F(x)=x^{3/2}$ ($0<x\le 1$) $G(y)=P(Y\le y)$ $= P(1/\sqrt{X}\le y)$ $= P(X\ge 1/y^2)$ $= 1- F(1/y^2)$ $= 1 - 1/y^3$ for $y\ge 1$ ($G(y)=0$ otherwise) $g(y) = G'(y) = 3/y^4$ $y\ge 1$, AG ($= 0$ otherwise)	B1 M1 M1 A1,B1 M1A1 **[7]**	B1 for $y \ge 1$ seen anywhere. M1 for differentiating Allow from $-1/y^3$	
(ii)	(α) $\int_1^{\infty} \frac{3}{y^2}\,dy = 3$ (β) $\int_0^1 \frac{3}{2\sqrt{x}}\,dx = 3$	M1A1 M1A1 **[2]**	Ft range from (i) for M1, but not $\int_0^1 \frac{3}{y^2}dy$	

Q8, (Jun 2013, Q5)

(i)	f is non-negative over [1, e] Attempt to show area, between 1 and e, = 1 $\int_1^e \ln y\,dy = y\ln y - \int dy$ $= \left[y\ln y - y\right]_1^e$ $= (e\ln e - e) - (1\ln 1 - 1) = 1$	B1 M1 M1 A1 **[4]**	 Integrate by parts. Allow 1 error. cwo
(ii)	$F(y) = \int_1^y \ln t\,dt = \left[t\ln t - t\right]_1^y$ $= (y\ln y - y) - (1\ln 1 - 1)$ $= y\ln y - y + 1$ AG, over [1, e]	M1 A1 A1 **[3]**	Limits must be correct. If indef integral, must have '$+c$' and attempt to evaluate. $1\ln 1 - 1 + c = 0$ OR $e\ln e - e + c = 1$ Needs proper justification.
(iii)	$F(2.45) = 0.745$, $F(2.46) = 0.754$ and $0.745 < 0.75 < 0.754$ and result follows	M1 A1 **[2]**	or $1 - F = 0.255, 0.246$ or $y\ln y - y + 1 = 0.75$ oe

(iv)		$G(x) = P(X < x)$ $= P(\ln Y < x)$ $= P(Y < e^x)$ $= F(e^x)$ $= (e^x \ln e^x - e^x + 1)$ $\quad xe^x - e^x + 1$; over $[0,1]$	 M1 M1 A1;B1 **[4]**	Allow M0M1A1B1.

Q9, (Jun 2014, Q9)

(i)		$A = X(10 - X)$ Use CTS $A = 25 - (X - 5)^2$ AG	B1 M1 A1 **[3]**	from base x height or quadratic formula	 Allow verification.
(ii)		$f_X(x) = ½$	B1 **[1]**	Ignore range.	
(iii)		$F_X(x) = ½x$ $(F_A(a) =)\ P(A \le a) = P[X(10 - X) \le a]$ $= F_X(5 - \sqrt{(25 - a)})$ $= \frac{1}{2}(5 - \sqrt{25 - a})$ AG $0 \le A \le 16$ AG explained.	B1 M1 A1 A1 B1 **[5]**	Only if (ii) correct. Fully justified. eg x=2→a=16	 X(or x)≥5+√(25-a) is impossible. $F_A(16)=1$ is not enough.
(iv)		$f_A(a) = \frac{1}{4}(25 - a)^{-\frac{1}{2}}$	M1,A1 **[2]**	M1 for attempt at differentiation.	

Q10, (Jun 2016, Q8)

(i)		pdf of $R = \frac{1}{10}$ soi	B1	Method based on int. by subn. 1st B1 as main scheme. $\int_0^{10} \frac{1}{10} dr = 1$ M1* $\frac{dA}{dr} = 8\pi r$ *M1 $\int_0^{400\pi} \frac{1}{80\pi} \frac{dA}{\sqrt{\frac{A}{4\pi}}} = 1$ M1 pdf= $\frac{1}{40\sqrt{\pi a}}$ B1 attempt int to obtain cdf. M1	NOT $8\pi r$ alone
				$\frac{\sqrt{a}}{20\sqrt{\pi}}$ A1 final B1 for range as main scheme.	
		Cdf of $R = \frac{r}{10}$ allow $\frac{x}{10}$	B1		
		$[F_A(a) =]$ $P(A \le a) = P(4\pi R^2 \le a)$	M1		
		$P(R \le \sqrt{[a/4\pi]}) = F_R(\sqrt{[a/4\pi]})$	M1	allow M0M1 for eg πr^2 rearranged and substituted correctly.	
		$\frac{\sqrt{a}}{20\sqrt{\pi}}$ (SC B1 if obtained without M marks)	A1		
		$f_A(a) = \frac{1}{40\sqrt{a\pi}}$	M1A1	M1 for diffn.	
		$0 \le a \le 400\pi$	B1 **[8]**		