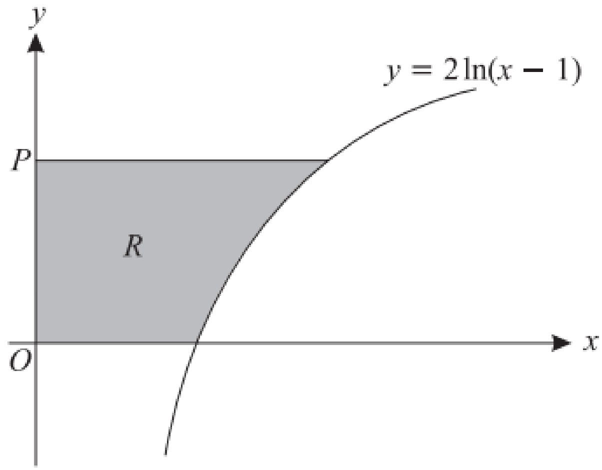
Volumes of Revolution

Q1, (OCR 4723, Jun 2006, Q9)



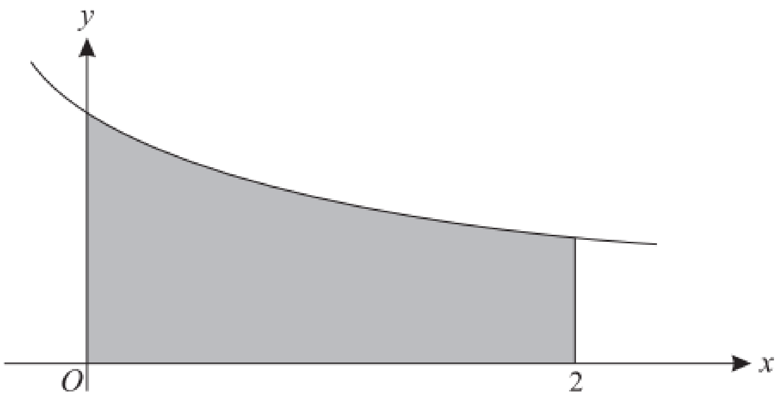
The diagram shows the curve with equation $y = 2\ln(x-1)$. The point P has coordinates $(0, p)$. The region R, shaded in the diagram, is bounded by the curve and the lines $x = 0$, $y = 0$ and $y = p$. The units on the axes are centimetres. The region R is rotated completely about the **y-axis** to form a solid.

(i) Show that the volume, V cm^3, of the solid is given by

$$V = \pi\left(e^{p} + 4e^{\frac{1}{2}p} + p - 5\right).$$ [8]

(ii) It is given that the point P is moving in the positive direction along the y-axis at a constant rate of 0.2 cm min^{-1}. Find the rate at which the volume of the solid is increasing at the instant when $p = 4$, giving your answer correct to 2 significant figures. [5]

Q2, (OVR 4723, Jan 2007, Q6)

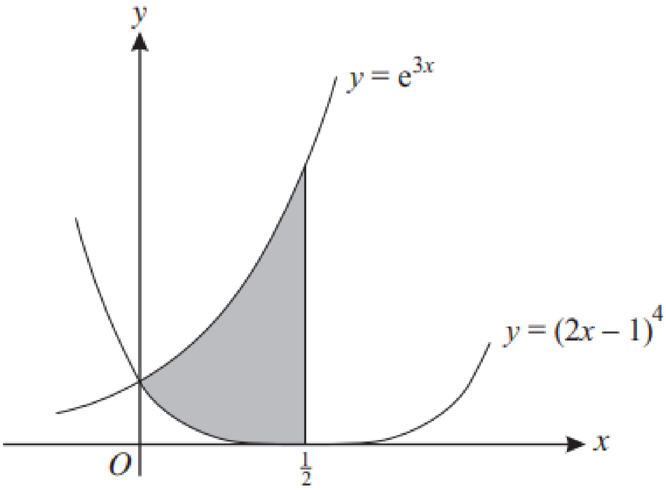


The diagram shows the curve with equation $y = \dfrac{1}{\sqrt{3x+2}}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$.

(i) Find the exact area of the shaded region. [4]

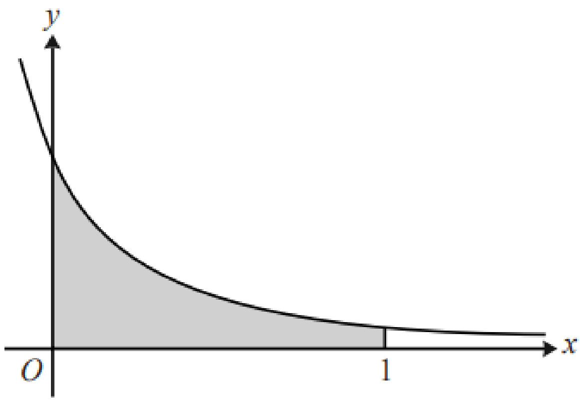
(ii) The shaded region is rotated completely about the x-axis. Find the exact volume of the solid formed, simplifying your answer. [5]

Q3, (Jun 2008, Q6)



The diagram shows the curves $y = e^{3x}$ and $y = (2x - 1)^4$. The shaded region is bounded by the two curves and the line $x = \frac{1}{2}$. The shaded region is rotated completely about the x-axis. Find the exact volume of the solid produced. [9]

Q4, (Jan 2012, Q2)



The diagram shows part of the curve $y = \frac{6}{(2x+1)^2}$. The shaded region is bounded by the curve and the lines $x = 0$, $x = 1$ and $y = 0$. Find the exact volume of the solid produced when this shaded region is rotated completely about the x-axis. **[5]**

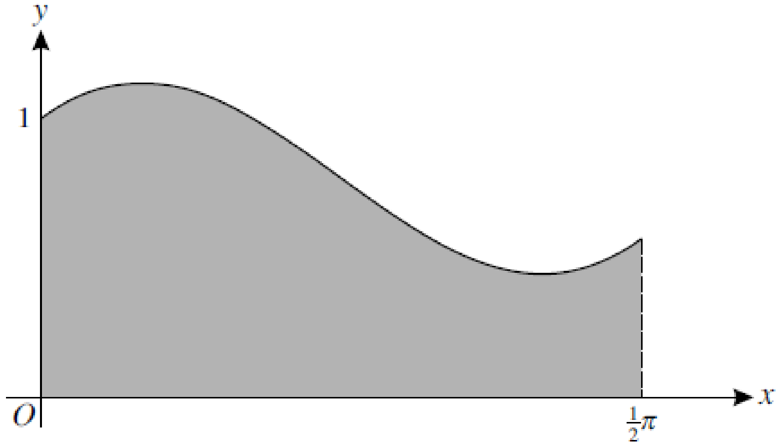
Q5, (OCR 4724, Jun 2007, Q3)

Find the exact volume generated when the region enclosed between the x-axis and the portion of the curve $y = \sin x$ between $x = 0$ and $x = \pi$ is rotated completely about the x-axis. [6]

Q6, (Jun 2010, Q9)

(i) Find $\int (x + \cos 2x)^2 \, dx$. **[9]**

(ii)

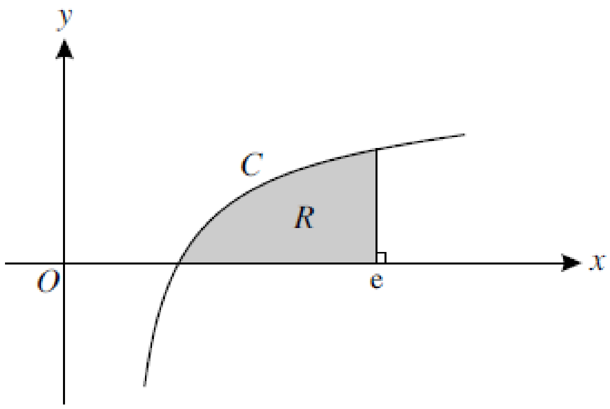


The diagram shows the part of the curve $y = x + \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The shaded region bounded by the curve, the axes and the line $x = \frac{1}{2}\pi$ is rotated completely about the x-axis to form a solid of revolution of volume V. Find V, giving your answer in an exact form. **[4]**

Q7, (Jun 2011, Q9)

(i) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$. **[3]**

(ii)



In the diagram, C is the curve $y = \ln x$. The region R is bounded by C, the x-axis and the line $x = e$.

(a) Find the exact volume of the solid of revolution formed by rotating R completely about the x-axis. **[6]**

(b) The region R is rotated completely about the y-axis. Explain why the volume of the solid of revolution formed is given by

$$\pi e^2 - \pi \int_0^1 e^{2y} \, dy,$$

and find this volume. **[4]**

Q8, (Edexcel 6666, Jun 2009, Q8)

(a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \, d\theta$.

(2)

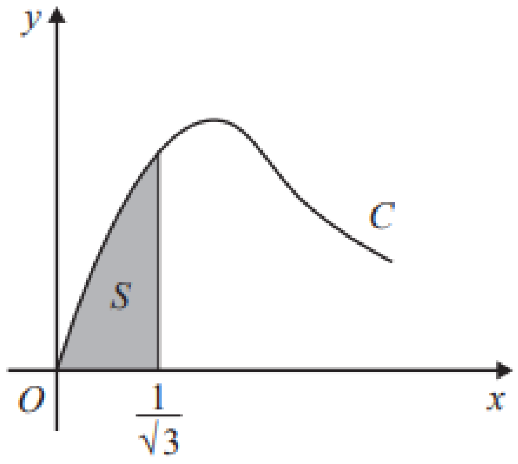


Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$$

where k is a constant.

(5)

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

Q9, (Edexcel 6666, Jan 2011, Q6)

The curve C has parametric equations

$$x = \ln t, \qquad y = t^2 - 2, \qquad t > 0$$

Find

(a) an equation of the normal to C at the point where $t = 3$,

(6)

(b) a cartesian equation of C.

(3)

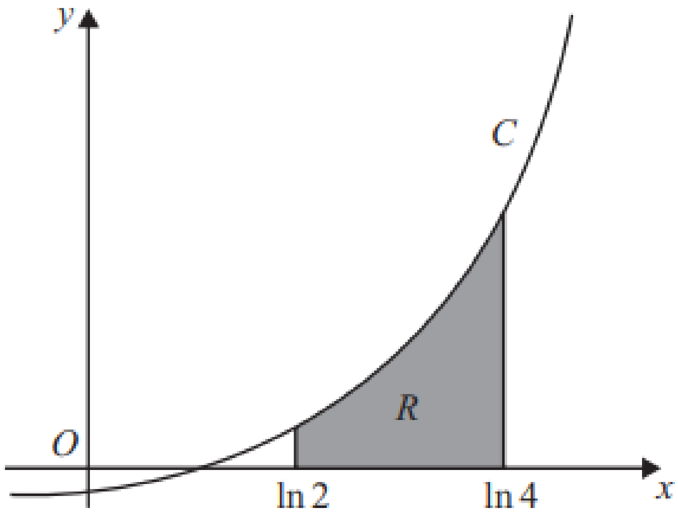


Figure 1

The finite area R, shown in Figure 1, is bounded by C, the x-axis, the line $x = \ln 2$ and the line $x = \ln 4$. The area R is rotated through $360°$ about the x-axis.

(c) Use calculus to find the exact volume of the solid generated.

(6)

Q10, (Edexcel 6666, Jun 2011, Q7)

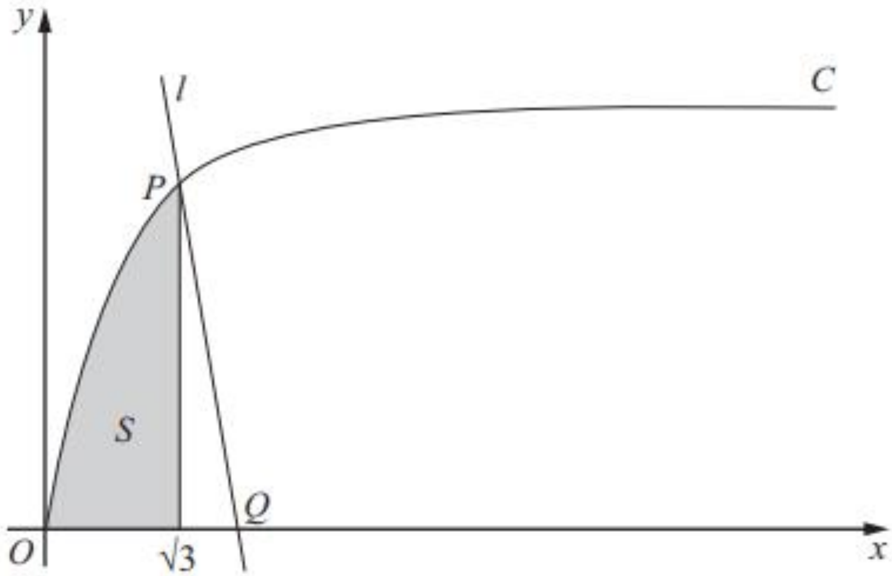


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)