Volumes of Revolution

Q1, (OCR 4723, Jun 2006, Q9)

(i)	Attempt to express *x* in terms of *y*	*M1		obtaining two terms
	Obtain $x = e^{\frac{1}{2}y} + 1$	A1		or equiv
	State or imply volume involves $\int \pi x^2$	B1		
	Attempt to express x^2 in terms of *y*	*M1		dep *M; expanding to produce at least 3 terms
	Obtain $k\int (e^y + 2e^{\frac{1}{2}y} + 1)\,dy$	A1		any constant *k* including 1; allow if d*y* absent
	Integrate to obtain $k(e^y + 4e^{\frac{1}{2}y} + y)$	A1		
	Use limits 0 and *p*	M1		dep *M *M; evidence of use of 0 needed
	Obtain $\pi(e^p + 4e^{\frac{1}{2}p} + p - 5)$	A1	8	**AG**; necessary detail required
(ii)	State or imply $\frac{dp}{dt} = 0.2$	B1		maybe implied by use of 0.2 in product
	Obtain $\pi(e^p + 2e^{\frac{1}{2}p} + 1)$ as derivative of *V*	B1		
	Attempt multiplication of values or expressions for $\frac{dp}{dt}$ and $\frac{dV}{dp}$	M1		
	Obtain $0.2\pi(e^4 + 2e^2 + 1)$	A1√		following their $\frac{dV}{dp}$ expression
	Obtain 44	A1	5	or greater accuracy

Q2, (OCR 4723, Jan 2007, Q6)

(i)	Obtain integral of form $k(3x+2)^{\frac{3}{2}}$	M1		any constant *k*
	Obtain correct $\frac{2}{9}(3x+2)^{\frac{3}{2}}$	A1		or equiv
	Substitute limits 0 and 2 and attempt evaluation	M1		for integral of form $k(3x+2)^n$
	Obtain $\frac{2}{9}(8^{\frac{3}{2}} - 2^{\frac{3}{2}})$	A1	4	or exact equiv suitably simplified
(ii)	State or imply $\pi\int \frac{1}{3x+2}\,dx$ or unsimplified version	B1		allow if d*x* absent or wrong
	Obtain integral of form $k\ln(3x+2)$	M1		any constant *k* involving π or not
	Obtain $\frac{1}{3}\pi\ln(3x+2)$ or $\frac{1}{3}\ln(3x+2)$	A1		
	Show correct use of ln *a* – ln *b* property	M1		
	Obtain $\frac{1}{3}\pi\ln 4$	A1	5	or (similarly simplified) equiv

Q3, (OCR 4723, Jun 2008, Q6)

Integrate k_1e^{nx} to obtain $k_2 e^{nx}$	**M1**	any constants involving π or not; any n
Obtain correct indefinite integral of their k_1e^{nx}	**A1**	
Substitute limits to obtain $\frac{1}{6}\pi(e^3-1)$ or $\frac{1}{6}(e^3-1)$	**A1**	or exact equiv perhaps involving e^0
Integrate $k(2x-1)^n$ to obtain $k'(2x-1)^{n+1}$	**M1**	any constants involving π or not; any n
Obtain correct indefinite integral of their $k(2x-1)^n$	**A1**	
Substitute limits to obtain $\frac{1}{18}\pi$ or $\frac{1}{18}$	**A1**	or exact equiv
Apply formula $\int \pi y^2 dx$ at least once	**B1**	for $y=e^{3x}$ and/or $y=(2x-1)^4$
Subtract, correct way round, attempts at volumes	**M1**	allow with π missing but must involve y^2
Obtain $\frac{1}{6}\pi e^3 - \frac{2}{9}\pi$	**A1**	or similarly simplified exact equiv
	9	

Q4, (OCR 4723, Jan 2012, Q2)

State $2\ln x$	B1	may be implied by immediate use of limits
Use both relevant logarithm properties correctly	M1	either or both may be implied, eg by $2\ln\sqrt{6} = \ln 6$ or by $\ln 6 - \ln 2 = \ln 3$
Obtain $\ln 3$	A1	AG; with at least one property shown explicitly
	[3]	
State volume is $\int \frac{36\pi}{(2x+1)^4}dx$	B1	or equiv in terms of x; no need for limits; condone absence of dx; condone absence of π here if it appears later in solution (even as part of a wrong answer)
Obtain integral of form $k(2x+1)^n$	M1	for any $n \le -1$; with or without π; or ku^n following substitution; allow if $n=-5$; allow M1 if one slight slip occurs in $(2x+1)$
Obtain $-6\pi(2x+1)^{-3}$ or $-6(2x+1)^{-3}$	A1	or (unsimplified) equiv
Substitute correct limits and subtract	M1	the correct way round for integral of form $k(2x+1)^{-3}$; allow if one slight slip occurs in $(2x+1)$; not earned if limit 0 leads to $\ldots - 0$
Obtain $\frac{52}{9}\pi$	A1	or similarly simplified exact equiv
	[5]	

Q5, (OCR 4724, Jun 2007, Q3)

Volume = $(k)\int_0^{\pi} \sin^2 x\,(\mathrm{d}x)$	B1		where $k = \pi, 2\pi$ or 1; limits necessary
Suitable method for integrating $\sin^2 x$	*M1		eg $\int +/-1+/-\cos 2x\,(\mathrm{d}x)$ or single integ by parts & connect to $\int \sin^2 x\,(\mathrm{d}x)$
$\int \sin^2 x\,(\mathrm{d}x) = \frac{1}{2}\int 1-\cos 2x\,(\mathrm{d}x)$	A1		or $-\sin x\cos x + \int \cos^2 x(\mathrm{d}x)$
$\int \cos 2x\,(\mathrm{d}x) = \frac{1}{2}\sin 2x$	A1		or $-\sin x\cos x + \int 1-\sin^2 x\,(\mathrm{d}x)$
Use limits correctly	dep*M1		
Volume = $\frac{1}{2}\pi^2$ WWW Exact answer	A1	**6**	**Beware**: wrong working leading to $\frac{1}{2}\pi^2$
			6

Q6, (OCR 4724, Jun 2010, Q9)

(i)	Attempt to multiply out $(x+\cos 2x)^2$	M1	Min of 2 correct terms
	<u>Finding $\int 2x\cos 2x\,dx$</u>		
	Use $u=2x, dv=\cos 2x$	M1	1^{st} stage $f(x)+/-\int g(x)dx$
	1^{st} stage $x\sin 2x-\int \sin 2x\,dx$	A1	
	$\therefore \int 2x\cos 2x\,dx = x\sin 2x+\frac{1}{2}\cos 2x$	A1	
	<u>Finding $\int \cos^2 2x\,dx$</u>		
	Change to $k\int +/-1+/-\cos 4x\,dx$	M1	where $k=\frac{1}{2}$, 2 or 1
	Correct version $\frac{1}{2}\int 1+\cos 4x\,dx$	A1	
	$\int \cos 4x\,dx = \frac{1}{4}\sin 4x$	B1	seen anywhere in this part
	Result $=\frac{1}{2}x+\frac{1}{8}\sin 4x$	A1	
	(i) ans $=\frac{1}{3}x^3+x\sin 2x+\frac{1}{2}\cos 2x+\frac{1}{2}x+\frac{1}{8}\sin 4x\ (+c)$	A1 **9**	Fully correct
(ii)	$V=\pi\int_0^{\frac{1}{2}\pi}(x+\cos 2x)^2\ (dx)$	M1	
	Use limits 0 & $\frac{1}{2}\pi$ correctly on their **(i)** answer	M1	
	(i) correct value $=\frac{1}{24}\pi^3-\frac{1}{2}+\frac{1}{4}\pi-\frac{1}{2}$	A1	
	Final answer $=\pi\left(\frac{1}{24}\pi^3+\frac{1}{4}\pi-1\right)$	A1 **4**	**c.a.o.** No follow-through
		13	

Q7, (OCR 4724, Jun 2011, Q9)

(i) Treat $x \ln x$ as a product	M1	If $\int \ln x$, use parts $u = \ln x$, $dv = 1$
Obtain $x.\frac{1}{x} + \ln x$	A1	$x \ln x - \int 1\, dx = x \ln x - x$
Show $x.\frac{1}{x} + \ln x - 1 = \ln x$ WWW **AG**	A1 **3**	And state given result
(ii)(a) Part (a) is mainly based on the indef integral $\int (\ln x)^2\, dx$		
[A candidate stating e.g. $\int (\ln x)^2 dx = \int 2 \ln x\, dx$ or $= \int (\ln x - x)^2 dx$ **is awarded 0 for (ii)(a)]**		
Correct use of $\int \ln x\, dx = x \ln x - x$ anywhere in this part	B1	Quoted from (i) or derived
Use integ by parts on $\int (\ln x)^2\, dx$ with $u = \ln x, dv = \ln x$	M1	or $u = (\ln x)^2, dv = 1$
[For 'integration by parts, candidates must get to a 1st stage with format $f(x) +/- \int g(x) dx$]		
1st stage = $\ln x(x \ln x - x) - \int \frac{1}{x}(x \ln x - x) dx$ soi	A1	$x(\ln x)^2 - \int x.\frac{2}{x} \ln x\, dx$
2nd stage = $x(\ln x)^2 - 2x \ln x + 2x$ AEF (unsimplified)	A1	
$\therefore$ Value of definite integral between 1 & e $= e - 2$ cao	A1	Use limits on 2nd stage & produce cao
Volume = $\pi(e-2)$ ISW	A1 **6**	Answer as decimal value (only) $\rightarrow$ A0
Alternative method when subst. $u = \ln x$ used		
Attempt to connect dx and du	M1	
Becomes $\int u^2 e^u\, du$	A1	
First stage $u^2 e^u - \int 2u\, e^u du$	A1	
Third stage $(u^2 - 2u + 2)e^u$	A1	
Final A1 A1 available as before		
(b) Indication that reqd vol = vol cylinder – vol inner solid	M1	
Clear demonstration of either vol of cylinder being πe^2 (including reason for height $= \ln e$) or rotation of $x = e$ about the y-axis (including upper limit of $y = \ln e$)	A1	Could appear as $\pi \int_0^1 e^2\, dy$
$(\pi) \int x^2\, dy = (\pi) \int e^{2y} dy$	B1	
$\frac{\pi(e^2 + 1)}{2}$ or 13.2 or 13.18 or better	B1 **4**	May be from graphical calculator
	13	

Possible helpful points

1. M is Method; does the candidate know what he/she should be doing? It does not ask how accurate it is.. e.g. in Qu.4, a candidate saying $\frac{dx}{d\theta} = -\frac{1}{3}\cos\theta$ is awarded M1.
2. When checking if decimal places are acceptable, accept both rounding & truncation.
3. In general we ISW unless otherwise stated.
4. The symbol √ is sometimes used to indicate 'follow-through' in this scheme.

Q8, (Edexcel 6666, Jun 2009, Q8)

(a)	$\int \sin^2\theta \, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1	(2)
(b)	$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$		
	$\pi\int y^2 \, dx = \pi\int y^2 \frac{dx}{d\theta} d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta \, d\theta$	M1 A1	
	$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta} d\theta$	M1	
	$= 16\pi\int \sin^2\theta \, d\theta \qquad k = 16\pi$	A1	
	$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1	(5)
	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta\right)$		
(c)	$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1	
	$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ Use of correct limits	M1	
	$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3} \qquad p = \frac{4}{3},\ q = -2$	A1	(3)
			[10]

Q9, (Edexcel 6666, Jan 2011, Q6)

(a)	$\frac{dx}{dt}=\frac{1}{t}, \quad \frac{dy}{dt}=2t$	
	$\frac{dy}{dx}=2t^2$	M1 A1
	Using $mm'=-1$, at $t=3$	
	$m'=-\frac{1}{18}$	M1 A1
	$y-7=-\frac{1}{18}(x-\ln 3)$	M1 A1 (6)
(b)	$x=\ln t \Rightarrow t=e^x$	B1
	$y=e^{2x}-2$	M1 A1 (3)
(c)	$V=\pi\int(e^{2x}-2)^2\,dx$	M1
	$\int(e^{2x}-2)^2\,dx=\int(e^{4x}-4e^{2x}+4)\,dx$	M1
	$=\frac{e^{4x}}{4}-\frac{4e^{2x}}{2}+4x$	M1 A1
	$\pi\left[\frac{e^{4x}}{4}-\frac{4e^{2x}}{2}+4x\right]_{\ln 2}^{\ln 4}=\pi\left[(64-32+4\ln 4)-(4-8+4\ln 2)\right]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6) **[15]**

Q10, (Edexcel 6666, Jun 2011, Q7)

(a)	$\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$		M1	
	$\theta = \frac{\pi}{3}$	awrt 1.05	A1	**(2)**
(b)	$\frac{dx}{d\theta} = \sec^2\theta, \quad \frac{dy}{d\theta} = \cos\theta$			
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta} \quad \left(= \cos^3\theta\right)$		M1 A1	
At P,	$m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$	Can be implied	A1	
Using $mm' = -1$,	$m' = -8$		M1	
For normal	$y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$		M1	
At Q, $y = 0$	$-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$			
leading to	$x = \frac{17}{16}\sqrt{3} \qquad \left(k = \frac{17}{16}\right)$	1.0625	A1	**(6)**
(c)	$\int y^2\,dx = \int y^2 \frac{dx}{d\theta}\,d\theta = \int \sin^2\theta \sec^2\theta\,d\theta$		M1 A1	
	$= \int \tan^2\theta\,d\theta$		A1	
	$= \int \left(\sec^2\theta - 1\right)d\theta$		M1	
	$= \tan\theta - \theta \quad (+C)$		A1	
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0 - 0)\right]$		M1	
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2 \qquad \left(p = 1, q = -\frac{1}{3}\right)$		A1	**(7)**
				[15]