Summations (Method Of Differences) Exam Questions (From OCR 4725)

Q1, (Jun 2005, Q5)

(i) Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}. \quad [2]$$

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots + \frac{1}{(n+1)(n+2)}. \quad [4]$$

(iii) Hence write down the value of $\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}$. [1]

Q2, (Jan 2007, Q8)

(i) Show that $(r+2)! - (r+1)! = (r+1)^2 \times r!$. [3]

(ii) Hence find an expression, in terms of n, for

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots + (n+1)^2 \times n!. \quad [4]$$

(iii) State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \ldots$$

converges. [1]

Q3, (Jun 2006, Q9)

(i) Use the method of differences to show that

$$\sum_{r=1}^{n} \{(r+1)^3 - r^3\} = (n+1)^3 - 1. \quad [2]$$

(ii) Show that $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$. [2]

(iii) Use the results in parts **(i)** and **(ii)** and the standard result for $\sum_{r=1}^{n} r$ to show that

$$3\sum_{r=1}^{n} r^2 = \tfrac{1}{2}n(n+1)(2n+1). \quad [6]$$

Q4, (Jun 2008, Q3)

(i) Show that $\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}.$$ [4]

Q5, (Jun 2011, Q7)

(i) Show that $\frac{1}{r-1} - \frac{1}{r+1} \equiv \frac{2}{r^2-1}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=2}^{n} \frac{2}{r^2-1}$. **[5]**

(iii) Find the value of $\sum_{r=1000}^{\infty} \frac{2}{r^2-1}$. **[3]**

Q6, (Jun 2012, Q8)

(i) Show that $\frac{1}{r} - \frac{1}{r+2} \equiv \frac{2}{r(r+2)}$. **[1]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{2}{r(r+2)}$. **[6]**

(iii) Given that $\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$, find the value of N. **[4]**

Q7, (Jun 2014, Q6)

(i) Show that $\frac{1}{r^2} - \frac{1}{(r+2)^2} \equiv \frac{4(r+1)}{r^2(r+2)^2}$. **[2]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=1}^{n} \frac{4(r+1)}{r^2(r+2)^2}$. **[6]**

(iii) Find $\sum_{r=5}^{\infty} \frac{4(r+1)}{r^2(r+2)^2}$, giving your answer in the form $\frac{p}{q}$ where p and q are integers. **[2]**

Q8, (Jun 2015, Q8)

(i) Show that $\frac{3}{r-1} - \frac{2}{r} - \frac{1}{r+1} \equiv \frac{4r+2}{r(r^2-1)}$. **[2]**

(ii) Hence find an expression, in terms of n, for $\sum_{r=2}^{n} \frac{4r+2}{r(r^2-1)}$. **[6]**

(iii) Hence find the value of $\sum_{r=4}^{\infty} \frac{4r+2}{r(r^2-1)}$. **[2]**

Q9, (Jun 2016,Q8)

(i) Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. **[1]**

(ii) Hence find $\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[6]**

(iii) Find $\sum_{r=n}^{\infty} \frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[3]**

Q10, (Jun 2017, Q7)

(i) Show that $\frac{1}{2r-1} - \frac{1}{2r+5} \equiv \frac{6}{(2r-1)(2r+5)}$. **[1]**

Hence find

(ii) $\sum_{r=2}^{30} \frac{6}{(2r-1)(2r+5)}$, giving your answer correct to 3 decimal places, **[5]**

(iii) $\sum_{r=2}^{\infty} \frac{6}{(2r-1)(2r+5)}$, giving your answer as a single fraction. **[1]**
