Summations (Method Of Differences) Exam Questions (From OCR 4725)

Q1, (Jun 2005, Q5)

(i) $\dfrac{(r+1)^2 - r(r+2)}{(r+2)(r+1)}$	M1		Show correct process for subtracting fractions
$\dfrac{1}{(r+1)(r+2)}$	A1	2	Obtain given answer correctly
(ii) *EITHER*			
$\dfrac{2}{3}-\dfrac{1}{2}+\dfrac{3}{4}-\dfrac{2}{3}\ \dots\ \dfrac{n+1}{n+2}-\dfrac{n}{n+1}$	M1		Express terms as differences using (i)
	A1		At least first two and last term correct
	M1		Show or imply that pairs of terms cancel
$\dfrac{n+1}{n+2}-\dfrac{1}{2}$	A1	4	Obtain correct answer in any form
OR	M2		State that $\sum_{r=1}^{n} u_r = f(n+1) - f(1)$
	A1A1		Each term correct
(iii) $\dfrac{1}{2}$	B1 ft	1	Obtain value from their sum to n terms
		7	

Q2, (Jan 2007, Q8)

(i)	M1		Factor of $r!$ or $(r+1)!$ seen
	A1		Factor of $(r+1)$ found
$(r+1)^2 r!$	A1	3	Obtain given answer correctly
(ii)	M1		Express terms as differences using (i)
	A1		At least 1st two and last term correct
	M1		Show that pairs of terms cancel
$(n+2)! - 2!$	A1	4	Obtain correct answer in any form
(iii)	B1ft	1	Convincing statement for non-converging, ft their (ii)
		8	

Q3, (Jun 2006, Q9)

(i)	M1 A1		Show that terms cancel in pairs Obtain given answer correctly
(ii)	M1 A1		Attempt to expand and simplify Obtain given answer correctly
(iii) $(n+1)^3 - 1 - \frac{3}{2}n(n+1) - n$	B1 B1 M1 M1 A1		Correct Σr stated $\Sigma\, 1 = n$ Consider sum of three separate terms on RHS Required sum is LHS – two terms Correct unsimplified expression
$\frac{1}{2}n(n+1)(2n+1)$	A1	2	Obtain given answer correctly

Q4, (Jun 2008, Q3)

(i) $\frac{r}{(r+1)!}$	**M1**	Common denominator of $(r+1)!$ or $r!(r+1)!$
	A1	Obtain given answer correctly
	$\boxed{2}$	
(ii) $1-\frac{1}{(n+1)!}$	**M1**	Express terms as differences using (i)
	A1	At least 1st two and last term correct
	M1	Show pairs cancelling
	A1	Correct answer a.e.f.
	$\boxed{4}$	

Q5, (Jun 2011, Q7)

(i)	B1	**1**	Obtain **given** answer correctly
(ii)	M1		Express at least 1st two and last two terms using (i)
	A1		1st two terms correct
	A1		Last two terms correct
	M1		Show that correct terms cancel
$\frac{3}{2}-\frac{1}{n}-\frac{1}{(n+1)}$	A1	**5**	Obtain correct answer, a.e.f. in terms of n
(iii)	B1ft		Sum to infinity stated or implied or start at 1000 as in (ii)
	M1		S_∞ – their (ii) with $n = 999$ or 1000 or show correct cancelling
$\frac{1999}{999000}$	A1	**3**	Obtain correct answer, a.e.f.
			(condone 0.002)
	$\boxed{9}$		

Q6, (Jun 2012, Q8)

(i)			B1 **[1]**	Show **given** answer correctly
(ii)		$1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$	M1 M1 A1 A1 M1 A1 **[6]**	Express terms as differences using (i) Attempt this for at least first 3 terms First 3 terms all correct Last 2 terms correct Show terms cancelling Obtain correct answer, must be in terms of n
(iii)		$\frac{3}{2}$ $N=4$	B1ft B1 M1 A1 **[4]**	State or use correct sum to infinity Their sum to infinity – their (ii) = $\frac{11}{30}$ Attempt to solve correct equation Obtain only $N=4$

Q7, (Jun 2014, Q6)

(i)			M1 A1 **[2]**	Combine with a correct denominator Obtain **given** answer correctly	
(ii)		$1+\frac{1}{4}-\frac{1}{(n+1)^2}-\frac{1}{(n+2)^2}$	M1 M1 A1 A1 M1 A1 **[6]**	Express as differences using **(i)** Attempt this for at least first 3 terms First 3 terms all correct Last 2 terms all correct Show correct cancelling Obtain correct answer i.s.w.	Final answer must be in terms of n
(iii)		$\frac{61}{900}$	M1 A1 **[2]**	Start differences at $n = 5$ or S_∞ - S_4 Obtain correct answer, **with no errors seen**	$1+\frac{1}{4}-(1-\frac{1}{4}-\frac{1}{5^2}-\frac{1}{6^2})$

Q8, (Jun 2015, Q8)

(i)			M1 A1 **[2]**	Use correct common denominator, numerator must be quadratic Obtain **given** result	
(ii)		$\frac{7}{2}-\frac{3}{n}-\frac{1}{n+1}$	M1 M1 A1 A1 M1 A1 **[6]**	Express terms as differences using **(i)** Attempt this for at least first 3 terms First 3 terms all correct Last 2 terms correct Show terms cancelling Obtain correct answer, must be in terms of n	Need not be tidied up
(iii)		$\frac{5}{4}$	M1 A1 **[2]**	Attempt to start summation at correct term Obtain correct answer from correct working	Could be $\Sigma_2^\infty - \Sigma_2^3$

Q9, (Jun 2016,Q8)

(i)	$\frac{2r+3-(2r+1)}{(2r+1)(2r+3)}$ *or* $\frac{2r+3-2r-1}{(2r+1)(2r+3)} = \frac{2}{(2r+1)(2r+3)}$	B1 **[1]**	Establish **given** result correctly, might be a "0 = 0" type verification, or partial fractions
(ii)	1/3 – 1/5 and 1/5 – 1/7 1/(2*n* + 1) – 1/(2*n* +3) $\frac{n}{3(2n+3)}$	M1 M1 A1 A1 M1 A1 **[6]**	Express terms as differences using (i) Attempt this for at least 1st two and last term First two terms correct, do not penalise missing factor of 2 Last term correct, do not penalise missing factor of 2, allow in terms of e.g. r or k Show correct cancelling Obtain a correct single fraction (denominator could be expanded) must be in terms of n. N.B. Be on look out for f(r) – f(r + 1) approach
(iii)	*Either* $\frac{1}{6} - \frac{n-1}{3(2n+1)}$ $\frac{1}{2(2n+1)}$ *Or* $\frac{1}{(2n+1)}$ $\frac{1}{2(2n+1)}$	M1 A1 A1 **[3]** M1 A1 A1	Use sum to ∞ – sum to *n* -1 or *n* Obtain correct unsimplified expression, do not penalise missing factor of 2 Obtain a correct single fraction (denominator could be expanded) Start differences at *r* = *n* Obtain correct remaining term Obtain a correct single fraction (denominator could be expanded)

Q10, (Jun 2017, Q7)

(i)	$\frac{2r+5-(2r-1)}{(2r-1)(2r+5)}$ or $\frac{2r+5-2r+1}{(2r-1)(2r+5)}$ must be seen	B1 **[1]**	Derive **given** result correctly (or use complete partial fractions method) **N.B.** 1st expression could be 2 separate fractions
(ii)	*Either* 0.629 *Or* $\sum_{2}^{30} \frac{1}{2r-1} = 1.68237..$ $\sum_{2}^{30} \frac{1}{2r+5} = 1.05383..$ 0.629	M1* A1 A1 DM1 A1 **[5]** M1 A1 A1 M1 A1	Express at least 2 terms as differences using (i), could start at *r* = 1 Obtain $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}$, obtain $-\frac{1}{61}, -\frac{1}{63}, -\frac{1}{65}$ these may be unsimplified Show correct cancelling (and subtraction of 1st term if appropriate) Obtain correct answer Attempt to find sum of 2 separate series, could start at *r* = 1 Obtain correct answers, must be at least 4 decimal places Subtract their values Obtain correct answer
(iii)	71/105	B1 **[1]**	Obtain correct answer