Polar Coordinates (From OCR 4726)

Q1, (Jan 2006, Q8i,iii,iv)

The equation of a curve, in polar coordinates, is

$$r = 1 + \cos 2\theta, \quad \text{for } 0 \leqslant \theta < 2\pi.$$

(i) State the greatest value of r and the corresponding values of θ. [2]

(iii) Find the exact area enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$. [5]

(iv) Find, in simplified form, the cartesian equation of the curve. [4]

Q2, (Jun 2006, Q7ii-iv)

The equation of a curve, in polar coordinates, is

$$r = \sqrt{3} + \tan\theta, \quad \text{for } -\tfrac{1}{3}\pi \leqslant \theta \leqslant \tfrac{1}{4}\pi.$$

(ii) State the greatest value of r and the corresponding value of θ. [2]

(iii) Sketch the curve. [2]

(iv) Find the exact area of the region enclosed by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{4}\pi$. [5]

Q3, (Jan 2007, Q9)

The equation of a curve, in polar coordinates, is

$$r = \sec\theta + \tan\theta, \quad \text{for } 0 \leqslant \theta \leqslant \tfrac{1}{3}\pi.$$

(i) Sketch the curve. [2]

(ii) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [6]

(iii) Find a cartesian equation of the curve. [3]

Q4, (Jan 2011, Q7iii)

A curve has polar equation $r = 1 + \cos 3\theta$, for $-\pi < \theta \leqslant \pi$.

(iii) Find the exact value of the area of the region enclosed by the curve between $\theta = -\frac{1}{3}\pi$ and $\theta = \frac{1}{3}\pi$. [5]

Q5, (Jun 2013, Q8)

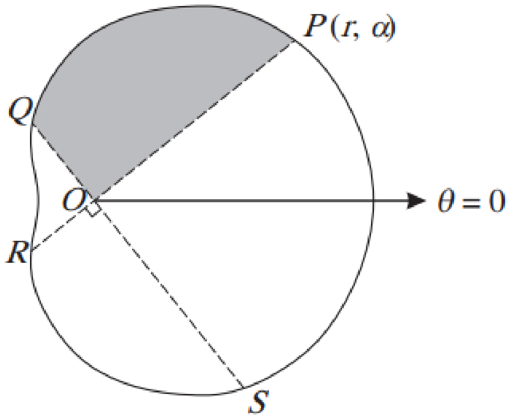
The equation of a curve is $x^2 + y^2 - x = \sqrt{x^2 + y^2}$.

(i) Find the polar equation of this curve in the form $r = f(\theta)$. [3]

(ii) Sketch the curve. [2]

(iii) The line $x + 2y = 2$ divides the region enclosed by the curve into two parts. Find the ratio of the two areas. [6]

Q6, (Jan 2009, Q7)



The diagram shows the curve with equation, in polar coordinates,

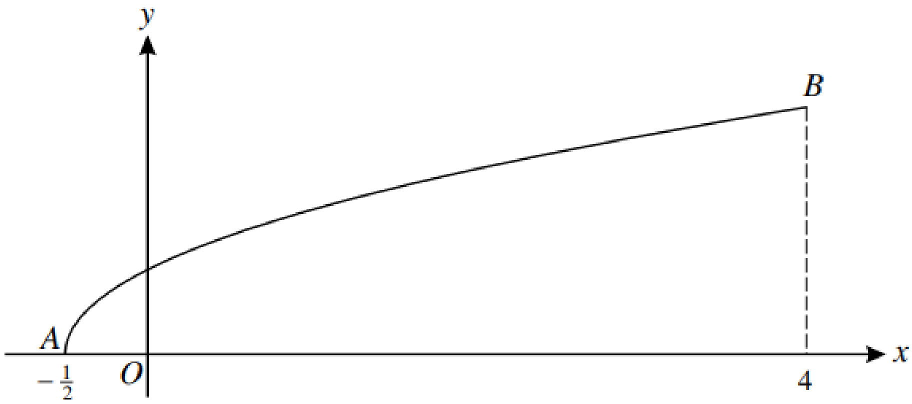
$$r = 3 + 2\cos\theta, \quad \text{for } 0 \leqslant \theta < 2\pi.$$

The points P, Q, R and S on the curve are such that the straight lines POR and QOS are perpendicular, where O is the pole. The point P has polar coordinates (r, α).

(i) Show that $OP + OQ + OR + OS = k$, where k is a constant to be found. **[3]**

(ii) Given that $\alpha = \frac{1}{4}\pi$, find the exact area bounded by the curve and the lines OP and OQ (shaded in the diagram). **[5]**

Q7, (Jun 2010, Q9)



The diagram shows the curve with equation $y = \sqrt{2x+1}$ between the points $A\,(-\frac{1}{2}, 0)$ and $B\,(4, 3)$.

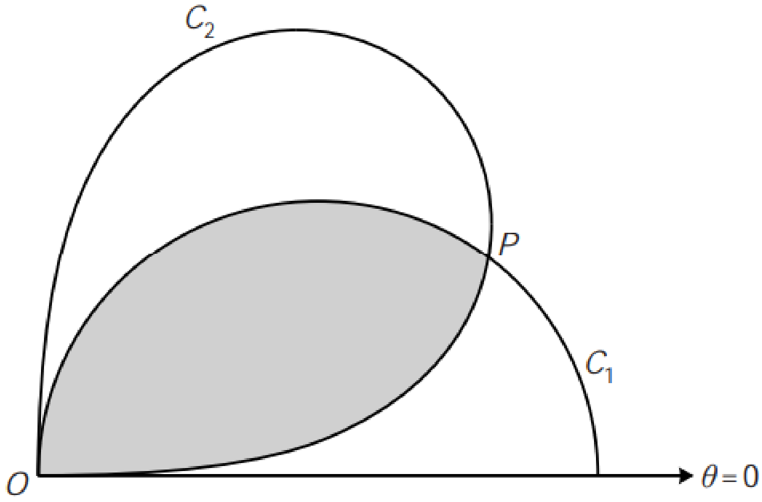
(i) Find the area of the region bounded by the curve, the x-axis and the line $x = 4$. Hence find the area of the region bounded by the curve and the lines OA and OB, where O is the origin. **[4]**

(ii) Show that the curve between B and A can be expressed in polar coordinates as

$$r = \frac{1}{1 - \cos\theta}, \quad \text{where } \tan^{-1}\left(\tfrac{3}{4}\right) \leqslant \theta \leqslant \pi. \qquad \textbf{[5]}$$

(iii) Deduce from parts **(i)** and **(ii)** that $\displaystyle\int_{\tan^{-1}\left(\frac{3}{4}\right)}^{\pi} \operatorname{cosec}^4\left(\tfrac{1}{2}\theta\right)\,d\theta = 24.$ **[4]**

Q8, (Jan 2012, Q8)



The diagram shows two curves, C_1 and C_2, which intersect at the pole O and at the point P. The polar equation of C_1 is $r = \sqrt{2}\cos\theta$ and the polar equation of C_2 is $r = \sqrt{2\sin 2\theta}$. For both curves, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. The value of θ at P is α.

(i) Show that $\tan\alpha = \frac{1}{2}$. **[2]**

(ii) Show that the area of the region common to C_1 and C_2, shaded in the diagram, is $\frac{1}{4}\pi - \frac{1}{2}\alpha$. **[7]**

Q9, (Jun 2014, Q8)

A curve has polar equation $r = a(1 + \cos\theta)$, where a is a positive constant and $0 \leqslant \theta < 2\pi$.

(ii) Sketch the curve. **[2]**

(iii) Find the area enclosed by the curve. **[6]**

Q10, (Jun 2015, Q9)

The equation of a curve in polar coordinates is $r = 2\sin 3\theta$ for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(i) Sketch the curve. **[2]**

(ii) Find the area of the region enclosed by this curve. **[4]**

(iii) By expressing $\sin 3\theta$ in terms of $\sin\theta$, show that a cartesian equation for the curve is

$$(x^2 + y^2)^2 = 6x^2y - 2y^3.$$ **[5]**