Partial Fractions and Integration (From OCR 4726)

Q1, (Jan 2006, Q3)

Use $A/x + (Bx + C)/(x^2 + 2)$	B1	
Equate $x+6$ to $A(x^2 + 2) + (Bx+C)x$ (or equiv.)	M1√	Equate to their P.F. (e.g. if B = 0 or C = 0 used)
Use $x = 0$ or equiv. for A (or equate coeff.etc.)	M1√	Include cover-up
Correctly find one of B,C	A1	
Get A=3, B=-3,C=1	A1	

Q2, (Jun 2007, Q3)

(i) Express as $A/(x-1) + (Bx+C)/(x^2+9)$	M1	Allow C=0 here
Equate (x^2+9x) to $A(x^2+9) + (Bx+C)(x-1)$	M1√	May imply above line; on their P.F.
Sub. for x or equate coeff.	M1	Must lead to at least 3 coeff.; allow cover-up method for A
Get A=1, B=0,C=9	A1	cao from correct method
(ii) Get $A\ln(x-1)$	B1√	On their A
Get $C/3 \tan^{-1}(x/3)$	B1√	On their C; condone no constant; ignore any $B \neq 0$

Q3, (Jun 2008, Q1)

Write as $\frac{A}{x-2a}+\frac{Bx+C}{x^2+a^2}$	**M1**	Accept C=0
Get $2ax = A(x^2+a^2) + (Bx+C)(x-2a)$	**A1√**	Follow-on for C=0
Choose values of x and/or equate coeff.	**M1**	Must lead to at least one of their A,B,C
Get $A = {}^4/_5$, $B = -{}^4/_5$, $C = {}^2/_5a$	**A1**	For two correct from correct working only
	A1	For third correct
	5	

Q4, (Jun 2009, Q4)

Attempt to divide out.	M1	Or $A+B/(x-2)+(Cx(+D))/(x^2+4)$; allow A=1 and/or B=1 quoted
Get $x^3=$ $A(x-2)(x^2+4)+B(x^2+4)+(Cx+D)(x-2)$	M1	Allow √ mark from their Part Fract; allow D=0 but not C=0
State/derive/quote A=1	A1	
Use x values and/or equate coeff	M1	To potentially get all their constants
Get B=1, C=1, D=-2	A1	For one other correct from cwo
	A1	For all correct from cwo

Q5, (Jan 2010, Q6)

(i)	Attempt at correct form of P.F.	M1	Allow $Cx/(x^2+1)$ here; not $C = 0$
	Rewrite as 4= $A(1+x)(1+x^2) + B(1-x)(1+x^2) + (Cx+D)(1-x)(1+x)$	M1 √	From their P.F.
	Use values of x/equate coefficients	M1	
	Get $A = 1$, $B = 1$	A1	cwo
	Get $C = 0$, $D = 2$	A1	
			SC Use of cover-up rule for A,B M1
			If both correct A1 cwo
(ii)	Get $A\ln(1+x) - B\ln(1-x)$	M1	Or quote from List of Formulae
	Get $D\tan^{-1}x$	B1	
	Use limits in their integrated expressions	M1	
	Clearly get A.G.	A1	

Q6, (Jan 2012, Q3)

$\frac{2x^3+x+12}{(2x-1)(x^2+4)} \equiv A+\frac{B}{2x-1}+\frac{Cx+D}{x^2+4}$	B1	For correct form soi (A can be $Px+Q$, but not 0)
$2x^3+x+12 \equiv A(2x-1)(x^2+4)+B(x^2+4)+(Cx+D)(2x-1)$	M1	For multiplying out from their form
$A=1, B=3$	B1	For either A or B correct (dep on 1st B1)
$x^3: 2=2A \quad x^2: 0=-A+B+2C$	M1	For equating at least 2 coefficients (or substitute two values for x or one of each)
$x^1: 1=8A-C+2D \qquad x^0: 12=-4A+4B-D$		
$C=-1, D=-4$	A1A1	For C, D correct
$\Rightarrow 1+\frac{3}{2x-1}+\frac{-x-4}{x^2+4}$	A1	For correct expression **WWW**
	[7]	**SC4** $\Rightarrow \frac{3}{2x-1}+\frac{x^2-x}{x^2+4}$

ALT: Divide out as not proper		
$\Rightarrow 1+\frac{x^2-7x+16}{(2x-1)(x^2+4)}$	B1	Divide out
$=1+\frac{A}{2x-1}+\frac{Bx+C}{x^2+4}$	B1	Writing in this form including 1
$x^2-7x+16 \equiv A(x^2+4)+(Bx+C)(2x-1)$	M1	For multiplying out from their form
$x^2: 1=A+2B \qquad x: -7=-B+2C$	M1	For equating at least 2 coefficients (or substitute two values for x or one of each)
$1: 16=4A-C$		
$\Rightarrow A=3, B=-1, C=-4$	A1	B correct
	A1	C correct
$\Rightarrow 1+\frac{3}{2x-1}+\frac{-x-4}{x^2+4}$	A1	For correct expression **WWW**

Q7, (Jun 2016, Q2)

$$f(x)=\frac{x(x-1)}{(x+1)(x^2+1)}\equiv\frac{A}{(x+1)}+\frac{Bx+C}{(x^2+1)}$$

M1	Correct partial fractions

$$\Rightarrow A(x^2+1)+(Bx+C)(x+1)\equiv x(x-1)$$

For e.g. equate coefficients

M1	Dep on 1st M

$$\Rightarrow A+B=1,\quad B+C=-1,\quad A+C=0$$

$$\Rightarrow A=1, B=0, C=-1$$

$$\Rightarrow (f(x))=\frac{1}{(x+1)}-\frac{1}{(x^2+1)}$$

A1	Dep on both M marks.

$$\Rightarrow \int_0^1 f(x)dx=\int_0^1\left(\frac{1}{(x+1)}-\frac{1}{(x^2+1)}\right)dx$$

B1	ft for integrating 1st term correctly (A/(x + 1) A ≠0)

$$=\left[\ln(1+x)-\tan^{-1}x\right]_0^1=\ln 2-\frac{\pi}{4}$$

B1	ft for subsequent term(s) correctly
B1	in exact form as dep on both previous B marks
[6]	