Maclaurin Expansion Exam Questions MS (From OCR 4726)

Q1, (Jan 2006, Q1)

(i) Use standard $\ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}$

M1 Allow e.g. $3x^2$, 2! etc.

M1 Attempt to simplify $(3x)^2$ etc.

$= 3x - 9x^2/2 + 9x^3$

A1 cao

(ii) Produce $(1 + x + x^2/2)$

B1

M1 Mult. 2 reasonable attempts, each of 3 terms (non-zero)

Get $3x - 3x^2/2 + 6x^3$

A1√ From their series

SC M1 Reasonable attempt at diff. and replace $x = 0$ (2 correct)

M1√ Put their values into correct Maclaurin expansion

A1 cao

(Applies to either/both parts)

Q2, (Jan 2007, Q1)

(i) f(O) = In 3 f

B1

f'(0) = $^1/_3$

B1

f"(O) = -$^1/_9$ A.G.

B1 Clearly derived

(ii) Reasonable attempt at Maclaurin

Ml Form In3 + $ax + bx^2$, with a,b related to f " f'

$f(x) = \ln 3 + {}^1/_3x - {}^1/_{18}x^2$

Al √J On their values off' and f"

SR Use $\ln(3+x)$ = In3 + In(1 + $^1/_3$ x) Ml Use Formulae Book to get

In3 + Y3X - Y2(VJX)2 =

In3 + Y3X - 1/lgX2 Al

Q3, (Jan 2009, Q1)

(i)	Give $1 + 2x + (2x)^2/2$	M1	Reasonable 3 term attempt e.g. allow $2x^2/2$
	Get $1 + 2x + 2x^2$	A1	cao
			SC Reasonable attempt at f′(0) and f″(0) M1
			Get $1+2x+2x^2$ cao A1
(ii)	$\ln((1+2x+2x^2) + (1-2x+2x^2)) =$	M1	Attempt to sub for e^{2x} and e^{-2x}
	$\ln(2+4x^2) =$	A1√	On their part (i)
	$\ln 2 + \ln(1 + 2x^2)$	M1	Use of log law in reasonable expression
	$\ln 2 + 2x^2$	A1	cao
			SC Use of Maclaurin for f ′(x) and f″(x) M1
			One correct A1
			Attempt f(0), f ′(0) and f″(0) M1
			Get cao A1

Q4, (Jan 2010, Q2)

(i)	Find $f'(x) = 1/(1+(1+x)^2)$	M1	Quoted or derived; may be simplified or left as $\sec^2 y\, dy/dx = 1$
	Get $f(0) = \frac{1}{4}\pi$ and $f'(0) = \frac{1}{2}$	A1√	On their f′(0); allow f(0)=0.785 but not 45
	Attempt $f''(x)$	M1	Reasonable attempt at chain/quotient rule or implicit differentiation
	Correctly get $f''(0) = -\frac{1}{2}$	A1	A.G.
(ii)	Attempt Maclaurin as $af(0)+bf'(0)+cf''(0)$	M1	Using their f(0) and f′(0)
	Get $\frac{1}{4}\pi + \frac{1}{2}x - \frac{1}{4}x^2$	A1	Cao; allow 0.785

Q5, (Jan 2013, Q5)

(i)		$f'(x) = -\sin x.e^{-x} + \cos x.e^{-x}$ $\Rightarrow f'(0) = 1$ $f(0) = 0$	M1 A1 A1 **[3]**	Diffn using product correctly. For **both** values www	
(ii)		$f'(x) = \cos x.e^{-x} - \sin x.e^{-x} = \cos x.e^{x} - f(x)$ $f''(x) = -f'(x) - \cos x.e^{-x} - f(x)$ $= -f'(x) - f'(x) - f(x) - f(x)$ $f''(x) = -2f'(x) - 2f(x)$ OR $-2\cos x.e^{-x}$ Showing the two equal $f''(0) = -2$	M1 A1 A1 A1 **[4]**	Diffn	
(iii)		$f''(x) = -2f'(x) - 2f(x)$ $\Rightarrow f'''(x) = -2f''(x) - 2f'(x)$ **oe** $\Rightarrow f'''(0) = 4 - 2 = 2$	B1 B1 **[2]**	Not involving trig or exp fns	=-f''+2f Or 2f' + 4f
(iv)		$f(x) = x - x^2 + \dfrac{x^3}{3}$	M1 A1 **[2]**		
		Alternative: Write down correct series expansion for e^{-x} and sin$x$ and multiply	M1 A1		

Q6, (Jun 2013, Q3)

(i)	$\frac{dy}{dx}=\frac{1}{1-\left(\frac{1-x}{3+x}\right)^2}\times\frac{-(3+x)-(1-x)}{(3+x)^2}$	B1 M1 A1	Sight of standard diffn for $\tanh^{-1}x$ Fn of fn **and** diffn of quotient Soi correct quotient (i.e. correct expression for 2nd part)
	$\Rightarrow\frac{dy}{dx}=\left(\frac{-4}{(3+x)^2-(1-x)^2}\right)=\frac{k}{1+x}$	A1	
	$\Rightarrow\frac{dy}{dx}=\frac{-1}{2(1+x)}$	A1	Correct for y'
	$\Rightarrow\frac{d^2y}{dx^2}=\frac{1}{2(1+x)^2}$	A1 **[6]**	2nd diffn (NB AG)
(ii)	When $x=0$, $y=\tanh^{-1}\frac{1}{3}$ or $\frac{1}{2}\ln 2$ or $\ln\sqrt{2}$	B1	For 1st value (needs to be exact)
	$\frac{dy}{dx}=-\frac{1}{2}$ $\frac{d^2y}{dx^2}=\frac{1}{2}$	B1	For both
	$\Rightarrow y=\tanh^{-1}\frac{1}{3}+\left(-\frac{1}{2}\right)x+\left(\frac{1}{2}\right)\frac{x^2}{2}$	M1	Use of correct Maclaurin's series
	$=\tanh^{-1}\frac{1}{3}-\frac{1}{2}x+\frac{x^2}{4}$	A1 **[4]**	Accept 0.347

Q7, (Jun 2014, Q2)

(i)	$\ln\ 1+x\ = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...$ $\Rightarrow \ln\ 1+x^2\ = x^2-\frac{x^4}{2}+\frac{x^6}{3}-\frac{x^8}{4}...$ Validity: $-1\le x\le 1$ or $\lvert x\rvert\le 1$	**B1** **B1** **B1** **[3]**	2 or 3 terms correct unsimplified All terms correct	
(ii)	$\ln\ 1+x^2\ = x^2-\frac{x^4}{2}+\frac{x^6}{3}-\frac{x^8}{4}-...$ Substitute $x=\frac{1}{2}$ $\Rightarrow \ln\left(\frac{5}{4}\right)=\left(\frac{1}{2}\right)^2-\frac{1}{2}\left(\frac{1}{2}\right)^4+\frac{1}{3}\left(\frac{1}{2}\right)^6-\frac{1}{4}\left(\frac{1}{2}\right)^8+...$ $=\frac{1}{4}\left(1-\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1}{3}\left(\frac{1}{2}\right)^4-\frac{1}{4}\left(\frac{1}{2}\right)^6+...\right)$ $\Rightarrow\left(1-\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1}{3}\left(\frac{1}{2}\right)^4-\frac{1}{4}\left(\frac{1}{2}\right)^6+...\right)$ $=4\ln\left(\frac{5}{4}\right)$ **isw**	**M1** **A1** **[2]**	Sub $x=\frac{1}{2}$ into *their* ans to (i) Single ln expression	Alt: divide by x^2 then sub

Q8, (Jun 2015, Q2)

	$\ln(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} - \ldots$ $\sin x = x - \frac{x^3}{6} + \ldots$ $\ln(1+\sin x) = \left(x - \frac{x^3}{6}\right) - \frac{1}{2}\left(x - \frac{x^3}{6}\right)^2 + \frac{1}{3}\left(x - \frac{x^3}{6}\right)^3 - \ldots$ $= x - \frac{1}{2}x^2 + x^3\left(\frac{1}{3} - \frac{1}{6}\right)$ $= x - \frac{1}{2}x^2 + \frac{1}{6}x^3$	**B1** **B1** **M1** **A1**	Soi. Allow an expansion in x Soi For combining series, even if wrong. Must include at least the cubic bracket. Ignore further terms www accept 3! for 6	
		4		
	Alternative using Maclaurin general formula $f(x) = \ln(1+\sin x)$ $f(0) = 0$ $f'(x) = \frac{\cos x}{(1+\sin x)}$ $f'(0) = 1$ $f''(x) = \frac{-1}{(1+\sin x)}$ $f''(0) = -1$ $f'''(x) = \frac{\cos x}{(1+\sin x)^2}$ $f'''(0) = 1$ Maclaurin: $f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2} + \frac{f'''(0)x^3}{6}$ $\Rightarrow f(x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3$	**B1** **B1** **M1** **A1**	For $f'(x)$ For (not necessarily simplified) $f''(x)$ and $f''(0)$ www For correct formula up to 4th term and substituting *their* values Accept 3! for 6	

Q9, (Jun 2016, Q5)

(i)	$\frac{dy}{dx}=\frac{2}{1+4x^2}$ $\frac{d^2y}{dx^2}=-2\left(1+4x^2\right)^{-2}\times 8x=\frac{-16x}{\left(1+4x^2\right)^2}=-4x\left(\frac{dy}{dx}\right)^2$	**B1** **M1** **A1** **[3]**	For first diffn Diffn again and making comparison	
(ii)	When $x=0,\ y=0,\frac{dy}{dx}=2,\ \frac{d^2y}{dx^2}=0$ $\Rightarrow \frac{d^3y}{dx^3}+4\left(\frac{dy}{dx}\right)^2+8x\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}=0$ When $x=0,\ \frac{d^3y}{dx^3}+16=0$ $\Rightarrow (y)=2x-\frac{8x^3}{3}$	**B1** **M1** **A1** **A1** **[4]**	Soi by final answer Differentiate the equation given For – 16 www Final answer	See below for alternative differentiation. SC4 Final formula from formula book including sight of $(2x)^3$. SC2 if final form only seen (i.e. no working) SC0 if final form only and wrong
(iii)	$x=\frac{1}{2}\Rightarrow \tan^{-1}1=\frac{\pi}{4}$ In series $x=1-\frac{1}{3}=\frac{2}{3}$ $\Rightarrow$ Estimate for $\pi=\frac{8}{3}=2.666....$ which, correct to 1sf, $=3$	**B1** **B1** **[2]**	soi For showing to 1sf π = 3 which is correct but to 2sf π = 2.7 which is not. www	