Intersection OF Planes

Q1, (OCR 4727, Jan 2011, Q7)

Three planes Π_1, Π_2 and Π_3 have equations

$$\mathbf{r}.(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 5, \qquad \mathbf{r}.(\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 6, \qquad \mathbf{r}.(\mathbf{i} + 5\mathbf{j} - 12\mathbf{k}) = 12,$$

respectively. Planes Π_1 and Π_2 intersect in a line l; planes Π_2 and Π_3 intersect in a line m.

(i) Show that l and m are in the same direction. **[5]**

(ii) Write down what you can deduce about the line of intersection of planes Π_1 and Π_3. **[1]**

(iii) By considering the cartesian equations of Π_1, Π_2 and Π_3, or otherwise, determine whether or not the three planes have a common line of intersection. **[4]**

Q2 (OCR 4727, Jan 2013, Q1)

Two planes have equations

$$x + 2y + 5z = 12 \text{ and } 2x - y + 3z = 5.$$

(i) Find the acute angle between the planes. **[3]**

(ii) Find a vector equation of the line of intersection of the planes. **[4]**

Q3, (OCR 4727, Jun 2014, Q1)

(i) Find a vector equation of the line of intersection of the planes $2x + y - z = 4$ and $3x + 5y + 2z = 13$. **[4]**

(ii) Find the exact distance of the point (2, 5, –2) from the plane $2x + y - z = 4$. **[2]**

Q4, (OCR 4727, Jun 2016, Q6)

The planes Π_1 and Π_2 have equations

$$\mathbf{r}.\begin{pmatrix}1\\2\\1\end{pmatrix} = 3 \text{ and } \mathbf{r}.\begin{pmatrix}2\\1\\4\end{pmatrix} = 5$$

respectively. They intersect in the line l.

(i) Find cartesian equations of l. **[4]**

The plane Π_3 has equation $\mathbf{r}.\begin{pmatrix}1\\5\\-1\end{pmatrix} = 1$.

(ii) Show that Π_3 is parallel to l but does not contain it. **[3]**

(iii) Verify that $(2,0,1)$ lies on planes Π_1 and Π_3. Hence write down a vector equation of the line of intersection of these planes. **[3]**

Q5, (OCR 4727, Jun 2017, Q4)

A plane Π_1 passes through the points $(1,2,-1)$, $(2,-3,1)$ and $(-1,0,2)$.

(i) Show that the plane Π_1 has equation $11x+7y+12z = 13$. **[4]**

The plane Π_2 has equation $3x+y+z = 4$.

(ii) Find a vector equation of the line of intersection of Π_1 and Π_2. **[4]**

(iii) Find the acute angle between Π_1 and Π_2. **[2]**

Q6, (OCR 4757, Jun 2007, Q1)

Three planes P, Q and R have the following equations.

$$\begin{aligned} \text{Plane } P&: \quad 8x - y - 14z = 20 \\ \text{Plane } Q&: \quad 6x + 2y - 5z = 26 \\ \text{Plane } R&: \quad 2x + y - z = 40 \end{aligned}$$

The line of intersection of the planes P and Q is K.

The line of intersection of the planes P and R is L.

(i) Show that K and L are parallel lines, and find the shortest distance between them. [9]

(ii) Show that the shortest distance between the line K and the plane R is $5\sqrt{6}$. [3]

The line M has equation $\mathbf{r} = (\mathbf{i} - 4\mathbf{j}) + \lambda(5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$.

(iii) Show that the lines K and M intersect, and find the coordinates of the point of intersection. [7]

(iv) Find the shortest distance between the lines L and M. [5]

Q7, (OCR 4757, Jun 2009, Q1)

The point A $(-1, 12, 5)$ lies on the plane P with equation $8x - 3y + 10z = 6$. The point B $(6, -2, 9)$ lies on the plane Q with equation $3x - 4y - 2z = 8$. The planes P and Q intersect in the line L.

(i) Find an equation for the line L. **[5]**

(ii) Find the shortest distance between L and the line AB. **[6]**

The lines M and N are both parallel to L, with M passing through A and N passing through B.

(iii) Find the distance between the parallel lines M and N. **[5]**

The point C has coordinates $(k, 0, 2)$, and the line AC intersects the line N at the point D.

(iv) Find the value of k, and the coordinates of D. **[8]**
