Intersection OF Planes

Q1, (OCR 4727, Jan 2011, Q7)

(i)	$[1, 1, -2] \times [1, -1, 3] = (\pm)[1, -5, -2]$	M1 A1		For using **×** of direction vectors For correct direction
	$[1, -1, 3] \times [1, 5, -12] = (\pm)[-3, 15, 6]$	M1 A1		For using **×** of direction vectors For correct direction
	$[-3, 15, 6] = k[1, -5, -2] \Rightarrow$ parallel	A1	**5**	For argument completed **AG** ($k = -3$ not essential)
(ii)	Line of intersection is parallel to l and m	B1	**1**	For correct statement
(iii)	METHOD 1			
	$\left.\begin{array}{r} x+y-2z=5 \\ x-y+3z=6 \end{array}\right\}$ e.g. $z=0 \Rightarrow \left(\frac{11}{2}, -\frac{1}{2}, 0\right)$ on l	M1 A1		For attempt to find points on 2 lines For a correct point on one line
	$\left.\begin{array}{r} x-y+3z=6 \\ x+5y-12z=12 \end{array}\right\}$ e.g. $z=0 \Rightarrow (7, 1, 0)$ on m	A1		For a correct point on another line
	$\left.\begin{array}{r} x+y-2z=5 \\ x+5y-12z=12 \end{array}\right\}$ e.g. $z=0 \Rightarrow \left(\frac{13}{4}, \frac{7}{4}, 0\right)$ on l_3			
	Different points $\Rightarrow$ no common line of intersection	A1	**4**	For correct answer
	METHOD 2			
	$\left.\begin{array}{r} x+y-2z=5 \\ x-y+3z=6 \end{array}\right\}$ e.g. $\Rightarrow z = 11-2x,\ y = 27-5x$	M1 A1		For finding (e.g.) y and z in terms of x *OR* eliminating one variable For correct expressions *OR* equations
	LHS of eqn 3 = $x + (135-25x) - (132-24x) = 3 \neq 12$	A1		For obtaining a contradiction from 3rd equation
	$\Rightarrow$ no common line of intersection	A1		For correct answer
	METHOD 3			
	LHS $\Pi_3 = 3\Pi_1 - 2\Pi_2$	M2		For attempt to link 3 equations
	RHS $3\times5 - 2\times6 = 3 \neq 12$	A1		For obtaining a contradiction
	$\Rightarrow$ no common line of intersection	A1		For correct answer
	SR Variations on all methods may gain full credit			**SR** f.t. may be allowed from relevant working
			10	

Q2 (OCR 4727, Jan 2013, Q1)

(i)		$\cos\theta = \dfrac{\left\|\begin{pmatrix}1\\2\\5\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\3\end{pmatrix}\right\|}{\sqrt{1^2+2^2+5^2}\sqrt{2^2+(-1)^2+3^2}} = \dfrac{15}{\sqrt{30}\sqrt{14}}$ $\theta = 0.750$ or $43.0°$	M1 A1 A1 **[3]**	Accept unsimplified If zero, then **sc1** for $n_1.n_2 = 15$ seen	
(ii)		$\begin{pmatrix}1\\2\\5\end{pmatrix}\times\begin{pmatrix}2\\-1\\3\end{pmatrix}=\begin{pmatrix}11\\7\\-5\end{pmatrix}$ (eg) $x=0 \Rightarrow 2y+5z=12, -y+3z=5 \Rightarrow y=1, z=2$ $\mathbf{r}=\begin{pmatrix}0\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}11\\7\\-5\end{pmatrix}$	M1 A1 M1 A1 **[4]**	 oe vector form	M1 requires evidence of method for cross product or at least 2 correct values calculated or any valid point e.g.(-11/7, 0, 19/7) (22/5, 19/5, 0) Must have full equation including 'r ='
		Alternative: Find one point Find a second point and vector between points multiple of $\begin{pmatrix}11\\7\\-5\end{pmatrix}$ $\mathbf{r}=\begin{pmatrix}0\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}11\\7\\-5\end{pmatrix}$	M1 M1 A1 A1		
		Alternative: Solve simultaneously Point found Direction found $\mathbf{r}=\begin{pmatrix}0\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}11\\7\\-5\end{pmatrix}$	M1 A1 A1 A1	to at least expressions for x,y,z parametrically, or two relationship between 2 variables.	

Q3, (OCR 4727, Jun 2014, Q1)

(i)	$\begin{pmatrix}2\\1\\-1\end{pmatrix}\times\begin{pmatrix}3\\5\\2\end{pmatrix}=\begin{pmatrix}7\\-7\\7\end{pmatrix}=7\begin{pmatrix}1\\-1\\1\end{pmatrix}$	M1 A1		M1 requires evidence of method for cross product or at least 2 correct values calculated
	(eg) $z=0\Rightarrow 2x+y=4, 3x+5y=13\Rightarrow x=1, y=2$	M1		or any valid point e.g.(0, 3, -1), (3, 0, 2)
	$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1	oe vector form	Must have full equation including '**r** ='
	Alternative: Find one point	M1		
	Find a second point and vector between points	M1		
	multiple of $\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
	$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
	Alternative: Solve simultaneously	M1	to at least expressions for x,y,z parametrically, or two relationship between 2 variables.	
	Point **and** direction found	M1 A1		
	$\mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$	A1		
		[4]		
(ii)	$\frac{\lvert 2\times2+5--2-4\rvert}{\sqrt{2^2+1^2+1^2}}=\frac{7}{\sqrt{6}}$	M1 A1	Condone lack of absolute signs for M1 oe surd form. isw	2.86 with no workings scores M1
	Alternative: find parameter for perpendicular meets plane and use to find distance	M1 **[2]**	For complete method with calculation errors	look for $\lambda = -7/6$

Q4, (OCR 4727, Jun 2016, Q6)

(i)		$\begin{pmatrix}1\\2\\1\end{pmatrix}\times\begin{pmatrix}2\\1\\4\end{pmatrix}=\begin{pmatrix}7\\-2\\-3\end{pmatrix}$	M1A1		
		finds point on both planes	B1	e.g. $(0,1,1)$	or $\left(\frac{7}{3},\frac{1}{3},0\right)$ or $\left(\frac{7}{2},0,-\frac{1}{2}\right)$
		$\frac{x}{-7}=\frac{y-1}{2}=\frac{z-1}{3}$	A1 [4]	oe	
ALT		$x+2y+z=3$ $2x+y+4z=5$			
		$3x+7z=7$	M1	Attempts to find at least 1 equation	
		$2x+7y=7$	A1	2 correct equations	or $3y-2z=1$
		$\frac{x}{-7}=\frac{y-1}{2}=\frac{z-1}{3}$	M1A1 [4]	oe of the form f(x) = g(y)= h(z)	
(ii)		$\begin{pmatrix}-7\\2\\3\end{pmatrix}\cdot\begin{pmatrix}1\\5\\-1\end{pmatrix}=-7+10-3=0$	M1	For scalar product, either shows method or gives answer of zero	
		$\Rightarrow l \parallel \Pi_3$	A1	for A1 must have working out line for scalar product	
		$(0,1,1)$ is on line, but $\begin{pmatrix}0\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\5\\-1\end{pmatrix}=4\neq 1$ so not on plane	B1 [3]		
ALT		$x+5y-z=1$ $7\lambda+5(1-2\lambda)-(1-3\lambda)=1$	M1A1		
		$\Rightarrow 4=1$ inconsistent, so l is parallel and not on plane	A1 [3]		
(iii)		$2+2\times 0+1=3$ (so on Π_1)			must show working for at least one plane
		$\begin{pmatrix}2\\0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\5\\-1\end{pmatrix}=1$ (so on Π_3)	B1	Verify both	
		Line has equation $\mathbf{r}=\begin{pmatrix}2\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}-7\\2\\3\end{pmatrix}$	M1 A1 [3]	oe vector form in cartesian form M1 only	if cross product calculated incorrectly then M0A0

Q5, (OCR 4727, Jun 2017, Q4)

(i)	$\overrightarrow{AB} = \begin{pmatrix}2\\-3\\1\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}1\\-5\\2\end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix}-1\\0\\2\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}-2\\-2\\3\end{pmatrix}$	M1*	Any two vectors in plane	Third is $\begin{pmatrix}-1\\0\\2\end{pmatrix} - \begin{pmatrix}2\\-3\\1\end{pmatrix} = \begin{pmatrix}-3\\3\\1\end{pmatrix}$
	$\mathbf{n} = \begin{pmatrix}1\\-5\\2\end{pmatrix} \times \begin{pmatrix}-2\\-2\\3\end{pmatrix} = \begin{pmatrix}-11\\-7\\-12\end{pmatrix} = -\begin{pmatrix}11\\7\\12\end{pmatrix}$	*M1dep	Depends on using attempted vectors in plane Condone 1 incorrect element if no working.	**ALT** $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b} + t\boldsymbol{c}$ Then eliminates one parameter to form 2 equations
		A1	Any multiple – linked to second M1 only Condone omission of final minus sign in this argument	
	$11x + 7y + 12z = 11(1) + 7(2) + 12(-1)$ $11x + 7y + 12z = 13$	A1	Must show substitution or dot product www. Shown **ag.** Must have some reasoning e.g. AB and AC referenced or described as a vector in the plane, normal referenced**, r = a** + s**b** +t**c**	Then eliminates t to get plane (A2, with A1 awarded for each side of equation
		[4]		**SC4** or verifying that all three points lie on the given plane **and** checking for non-collinearity
(ii)	$\begin{pmatrix}11\\7\\12\end{pmatrix} \times \begin{pmatrix}3\\1\\1\end{pmatrix} = \begin{pmatrix}-5\\25\\-10\end{pmatrix} = -5\begin{pmatrix}1\\-5\\2\end{pmatrix}$	M1	Attempts cross product of correct vectors	
		A1	Any multiple	
	$x = 0 \Rightarrow y = 7, z = -3$	B1	Find a point on line	or $\begin{pmatrix}\frac{7}{5}\\0\\-\frac{1}{5}\end{pmatrix}$, or $\begin{pmatrix}\frac{3}{2}\\-\frac{1}{2}\\0\end{pmatrix}$, $\begin{pmatrix}1\\2\\-1\end{pmatrix}$, $\begin{pmatrix}2\\-3\\1\end{pmatrix}$
	$\mathbf{r} = \begin{pmatrix}0\\7\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\-5\\2\end{pmatrix}$	A1	Oe vector equation	
		[4]	**ALT 1:** Find a point on line M1 Find a second point and use to find direction of line M1, A1 Write equation A1	**A2:** Reduce 2 equations to single equation in 2 variables.M1 Write these 2 variables using a parameter. M1 Find third variable parametrically. A1 Write equation. A1

Q6, (OCR 4757, Jun 2007, Q1)

(i)	$\mathbf{d}_K = \begin{pmatrix} 8 \\ -1 \\ -14 \end{pmatrix} \times \begin{pmatrix} 6 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 33 \\ -44 \\ 22 \end{pmatrix}$ $\left[= 11\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} \right]$ $\mathbf{d}_L = \begin{pmatrix} 8 \\ -1 \\ -14 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 15 \\ -20 \\ 10 \end{pmatrix}$ $\left[= 5\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} \right]$ Hence K and L are parallel For a point on K, $z=0,\ x=3,\ y=4$ i.e. $(3,\ 4,\ 0)$ For a point on L, $z=0,\ x=6,\ y=28$ i.e. $(6,\ 28,\ 0)$	M1* A1* A1 M1*A1* A1*	Finding direction of K or L One direction correct * *These marks can be earned anywhere in the question* Correctly shown Finding one point on K or L *or* $(6,\ 0,\ 2)$ *or* $(0,\ 8,\ -2)$ *etc* *Or* $(27,\ 0,\ 14)$ *or* $(0,\ 36,\ -4)$ *etc*
	$\left[\begin{pmatrix} 6 \\ 28 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}\right] \times \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 24 \\ 0 \end{pmatrix} \times \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = \begin{pmatrix} 48 \\ -6 \\ -84 \end{pmatrix}$ Distance is $\dfrac{\sqrt{48^2+6^2+84^2}}{\sqrt{3^2+4^2+2^2}} = \dfrac{\sqrt{9396}}{\sqrt{29}} = 18$	M1 M1 A1 **9**	For $(\mathbf{b}-\mathbf{a})\times\mathbf{d}$ Correct method for finding distance
	OR $\begin{pmatrix} 6+3\lambda-3 \\ 28-4\lambda-4 \\ 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = 0$ M1 $-87+29\lambda=0,\quad \lambda=3$ M1 Distance is $\sqrt{12^2+12^2+6^2}=18$ A1		For $(\mathbf{b}+\lambda\mathbf{d}-\mathbf{a}).\mathbf{d}=0$ Finding λ, and the magnitude
(ii)	Distance from $(3,4,0)$ to R is $\left\lvert \dfrac{2\times3+4-0-40}{\sqrt{2^2+1^2+1^2}} \right\rvert$ $= \dfrac{30}{\sqrt6} = \dfrac{30\sqrt6}{6} = 5\sqrt6$	M1A1 ft A1 ag **3**	
(iii)	K, M intersect if $1+5\lambda = 3+3\mu$ (1) $-4-4\lambda = 4-4\mu$ (2) $3\lambda = 2\mu$ (3) Solving (2) and (3): $\lambda=4,\ \mu=6$ Check in (1): LHS $=1+20=21$, RHS $=3+18=21$ Hence K, M intersect, at $(21,\ -20,\ 12)$	M1 A1 ft M1M1 M1A1 A1 **7**	At least 2 eqns, different parameters Two equations correct Intersection correctly shown *Can be awarded after M1A1M1M0M0*
	OR M meets P when M1 $8(1+5\lambda)-(-4-4\lambda)-14(3\lambda)=20$ A1 M meets Q when $6(1+5\lambda)+2(-4-4\lambda)-5(3\lambda)=26$ A1 Both equations have solution $\lambda=4$ A1 Point is on P, Q and M; hence on K and M M2 Point of intersection is $(21,\ -20,\ 12)$ A1		Intersection of M with **both** P and Q

(iv)	$\left[\begin{pmatrix}6\\28\\0\end{pmatrix}-\begin{pmatrix}1\\-4\\0\end{pmatrix}\right]\cdot\left[\begin{pmatrix}3\\-4\\2\end{pmatrix}\times\begin{pmatrix}5\\-4\\3\end{pmatrix}\right]=\begin{pmatrix}5\\32\\0\end{pmatrix}\cdot\begin{pmatrix}-4\\1\\8\end{pmatrix}=12$	M1A1 ft M1	For evaluating $\mathbf{d}_L \times \mathbf{d}_M$ For $(\mathbf{b}-\mathbf{c}).(\mathbf{d}_L \times \mathbf{d}_M)$
	Distance is $\dfrac{12}{\sqrt{4^2+1^2+8^2}}=\dfrac{12}{9}=\dfrac{4}{3}$	A1 ft A1 **5**	Numerical expression for distance

Q7, (OCR 4757, Jun 2009, Q1)

(i)	Putting $x=0$, $-3y+10z=6$, $-4y-2z=8$ $y=-2$, $z=0$	M1 A1	Finding coords of a point on the line or $(2,\ 0,\ -1)$, $(1,\ -1,\ -\frac{1}{2})$ etc
	Direction is given by $\begin{pmatrix}8\\-3\\10\end{pmatrix}\times\begin{pmatrix}3\\-4\\-2\end{pmatrix}$	M1	or finding a second point
	$=\begin{pmatrix}46\\46\\-23\end{pmatrix}$	A1	
	Equation of L is $\mathbf{r}=\begin{pmatrix}0\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\2\\-1\end{pmatrix}$	A1 ft **5**	*Dependent on M1M1* Accept any form Condone omission of '**r** ='
(ii)	$\overline{\text{AB}}\times\mathbf{d}=\begin{pmatrix}7\\-14\\4\end{pmatrix}\times\begin{pmatrix}2\\2\\-1\end{pmatrix}=\begin{pmatrix}6\\15\\42\end{pmatrix}$ $\left[\ =3\begin{pmatrix}2\\5\\14\end{pmatrix}\ \right]$	M1 A2 ft	Evaluating $\overline{\text{AB}}\times\mathbf{d}$ Give A1 ft if just one error
	Distance is $\left[\begin{pmatrix}-1\\12\\5\end{pmatrix}-\begin{pmatrix}0\\-2\\0\end{pmatrix}\right].\hat{\mathbf{n}}=\dfrac{\begin{pmatrix}-1\\14\\5\end{pmatrix}.\begin{pmatrix}2\\5\\14\end{pmatrix}}{\sqrt{2^2+5^2+14^2}}$	M1 A1 ft	Appropriate scalar product Fully correct expression
	$=\dfrac{138}{15}=\dfrac{46}{5}=9.2$	A1 **6**	
(iii)	$\lvert\overline{\text{AB}}\times\mathbf{d}\rvert=\left\lvert\begin{pmatrix}6\\15\\42\end{pmatrix}\right\rvert=\sqrt{6^2+15^2+42^2}$	M1 M1	For $\lvert\overline{\text{AB}}\times\mathbf{d}\rvert$ Evaluating magnitude
	Distance is $\dfrac{\lvert\overline{\text{AB}}\times\mathbf{d}\rvert}{\lvert\mathbf{d}\rvert}=\dfrac{\sqrt{6^2+15^2+42^2}}{\sqrt{2^2+2^2+1^2}}$	M1A1 ft	*In this part, M marks are dependent on previous M marks*
	$=\dfrac{45}{3}=15$	A1 **5**	