Hyperbolic Functions Identities Exam Questions (From OCR 4726)

Q1, (Jan 2006, Q9)

(i) Correct def^{n} of cosh x and sinh x — B1,B1
Expand $2.\frac{1}{2}(e^{x} - e^{-x}).\frac{1}{2}(e^{x} + e^{-x})$ — M1 Reasonable attempt
Clearly get $\frac{1}{2}(e^{2x} - e^{-2x})$ to A.G. — A1

(ii) Attempt to diff. and solve $dy/dx = 0$ — M1 Reasonable attempt
Use (ii) to get A cosh x (B sinh x+ C)=0 — M1
Clearly see cosh $x > 0$ or similar for one useable factor only — B1
Attempt to solve sinh x = - C/B — M1 Quote or via e^{-x} correctly
Get $x = \ln((3+\sqrt{13})/2)$ — A1
Justify one answer only for sinh x = -C/B — B1
Accurate test for MINIMUM — B1 First or second diff^{l} test with numeric evidence
B1 Correct value(s) for min.

Q2, (Jun 2006, Q4)

(i) Correct definition of cosh x or cosh $2x$ — B1
Attempt to sub. in RHS and simplify — M1 or LHS if used
Clearly produce A.G. — A1

(ii) Write as quadratic in cosh x — M1 $(2\cosh^{2}x - 7\cosh x - 4 = 0)$
Solve their quadratic accurately — A1√ Factorise/quadratic formula
Justify one answer only — B1 State cosh $x \geq 1$/graph; allow ≥ 0
Give $\ln(4 + \sqrt{15})$ — A1 cao; any one of $\pm \ln(4 \pm \sqrt{15})$ or decimal equivalent of ln ( )

Q3, (Jan 2009, Q6)

(i)	Correct definitions used	B1	
	Attempt at $(e^{x}-e^{-x})^{2}/4 + 1$	M1	Allow $(e^{x}+e^{-x})^{2}+1$; allow /2
	Clearly derive A.G.	A1	
(ii)	Form a quadratic in sinh x	M1	
	Attempt to solve	M1	Factors or formula
	Get sinh x = -½ or 3	A1	
	Use correct ln expression	M1	On their answer(s) seen once
	Get $\ln(-\frac{1}{2}+\sqrt{5}/2)$ and $\ln(3+\sqrt{10})$	A1	

Q4, (Jun 2010, Q8)

(i) Use correct definition of cosh x	B1	
Attempt to cube their definition involving e^{x} and e^{-x} (or e^{2x} and e^{x})	M1	Must be 4 terms
Put their 4 terms into LHS and attempt to simplify	M1	
Clearly get A.G.	A1	
		SC Allow one B1 for correct derivation from $\cosh 3x = \cosh(2x+x)$
(ii) Rewrite as $k\cosh 3x = 13$	M1	
Use ln equivalent on $13/k$	M1	Allow $\pm\ln$ or $\ln(13/k \pm \sqrt{(13/k)^2 - 1})$ for their k or attempt to set up and solve quadratic via exponentials
Get $x = (\pm)\ \frac{1}{3}\ln 5$	A1	
Replace in cosh x for u	M1	
Use $e^{a\ln b} = b^{a}$ at least once	M1	
Get $\frac{1}{2}(5^{\frac{1}{3}} + 5^{-\frac{1}{3}})$	A1	

Q5, (Jun 2013, Q2)

(i)	$\cosh x = \frac{e^x + e^{-x}}{2}, \ \sinh x = \frac{e^x - e^{-x}}{2}$	B1	Correct formulae	
	$\Rightarrow \cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$	M1	Dealing with squaring correctly	Difference of squares can be used
	$= \frac{1}{4}\left(e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}\right) = \frac{1}{4}.4 = 1$	A1 **[3]**	**www** All steps seen	
(ii)	$\Rightarrow \cosh^2 x - 1 = 5\cosh x - 7$ $\Rightarrow \cosh^2 x - 5\cosh x + 6 = 0$ $\Rightarrow (\cosh x - 2)(\cosh x - 3) = 0$	M1 M1	Use **(i)** Attempt to solve quadratic	E.g. correct formula or expansion of their brackets gives 2 out of 3 terms correct
	$\Rightarrow \cosh x = 2, 3$	A1		
	$\Rightarrow x = \cosh^{-1} 2 = \pm \ln\left(2 \pm \sqrt{3}\right)$	A1	Use correct ln formula	Condone lack of ±
	and $x = \cosh^{-1} 3 = \pm \ln\left(3 \pm \sqrt{8}\right)$	A1 **[5]**	Use correct ln formula	Condone lack of ±

Q6, (Jan 2012, Q7)

(i)

$x = \sinh y = \dfrac{e^y - e^{-y}}{2}$

$\Rightarrow e^{2y} - 2xe^y - 1 = 0 \Rightarrow e^y = x \pm \sqrt{x^2+1}$

reject – sign as $e^y > 0 \Rightarrow y = \ln\left(x + \sqrt{x^2+1}\right)$

M1 For correct expression for sinh y and attempt to obtain quadratic

A1 For correct solution(s) for e^y

A1 For justification of + sign to **AG**

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Alt:

$\sinh y + \cosh y = e^y$

$\sinh y = x \Rightarrow \cosh y = \pm\sqrt{x^2+1}$

reject -ve sign as $e^y > 0$

$\Rightarrow e^y = x + \sqrt{x^2+1} \Rightarrow y = \ln\left(x + \sqrt{x^2+1}\right)$

(ii)

$\ln\left(x + \sqrt{x^2+1}\right) - \ln\left(x + \sqrt{x^2-1}\right) = \ln 2$

M1 For stating both ln expressions and attempting to exponentiate

Removing lns is not an attempt to exponentiate

$\Rightarrow \dfrac{x + \sqrt{x^2+1}}{x + \sqrt{x^2-1}} = 2$

$\Rightarrow \sqrt{x^2+1} - 2\sqrt{x^2-1} = x$

A1 For correct equation **AG**

M1 For attempting to square once

$\Rightarrow 4x^2 - 3 = 4\sqrt{x^4-1}$

A1 For a correct equation with $\sqrt{\ }$ as subject

$\Rightarrow 24x^2 = 25 \Rightarrow x = \dfrac{5}{\sqrt{24}} \left(= \dfrac{5}{12}\sqrt{6}\right)$

A1 For correct x and no others **isw**

[5]

Q8, (Jun 2015, Q1)

	$\tanh^{-1} x = y \Rightarrow x = \tanh y = \dfrac{e^{2y}-1}{e^{2y}+1}$	**M1**	**Oe**	A muddle of x and y unless recovered is M0.
	$(e^{2y}+1)x = e^{2y}-1$			
	$e^{2y}(1-x) = (1+x)$			
	$\Rightarrow e^{2y} = \dfrac{1+x}{1-x}$	**A1**	Correct expression for e^{2y} **oe**	
	$2y = \ln\left(\dfrac{1+x}{1-x}\right)$			
	$\left(y = \tanh^{-1} x\right) = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$	**A1**	**ag**	
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Q9, (Jun 2016, Q1)

(i)	$(e^{x}+e^{-x})^3 = e^{3x}+3e^{x}+3e^{-x}+e^{-3x}$	**M1**	Doing the expansion	
	$= (e^{3x}+e^{-3x})+3(e^{x}+e^{-x})$	**A1**		
	$\Rightarrow (2\cosh x)^3 = 2\cosh 3x + 6\cosh x$	**M1**	Relating cosh3*x* to exponentials correctly	
	$\Rightarrow 8\cosh^3 x = 2\cosh 3x + 6\cosh x$	**A1**		
	$\Rightarrow \cosh 3x = 4\cosh^3 x - 3\cosh x$	**[4]**		
(ii)	$\Rightarrow \cosh 3x = 4\cosh^3 x - 3\cosh x = 6\cosh x$	**M1**	Using result of (i)	
	$\Rightarrow 4\cosh^3 x = 9\cosh x$			
	$\Rightarrow \cosh^2 x = \dfrac{9}{4}$ since $\cosh x \neq 0$	**A1**	At least one rejection needs to be stated. Or cosh*x* ≥ 1	
	$\Rightarrow \cosh x = (\pm)\dfrac{3}{2}$ $\cosh x \neq -\dfrac{3}{2}$	**A1**		
	$\Rightarrow x = \pm\ln\left(\dfrac{3}{2}+\sqrt{\left(\dfrac{3}{2}\right)^2-1}\right)$	**A1**	A1 for each in exact form	
	$= \pm\ln\left(\dfrac{3}{2}+\dfrac{1}{2}\sqrt{5}\right)$ or $\ln\left(\dfrac{3}{2}\pm\dfrac{1}{2}\sqrt{5}\right)$	**A1**	Deduct from 5 marks 1 mark for additional incorrect answers	
		[5]		