First Order Differential Equations (From OCR 4727)

Q1, (Jun 2007, Q8)

(i) Find the general solution of the differential equation

$$\frac{dy}{dx} + y\tan x = \cos^3 x,$$

expressing y in terms of x in your answer. [8]

(ii) Find the particular solution for which $y = 2$ when $x = \pi$. [2]

Q2, (Jan 2008, Q5)

(i) Find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = \sin 2x,$$

expressing y in terms of x in your answer. [6]

In a particular case, it is given that $y = \frac{2}{\pi}$ when $x = \frac{1}{4}\pi$.

(ii) Find the solution of the differential equation in this case. [2]

(iii) Write down a function to which y approximates when x is large and positive. [1]

Q3, (Jun 2009, Q4) [Modified]

The differential equation

$$\frac{dy}{dx} + \frac{1}{1-x^2}y = (1-x)^{\frac{1}{2}}, \quad \text{where } |x| < 1,$$

can be solved by the integrating factor method.

(i) Use partial fractions or another appropriate method in order to show that the integrating factor can be written as $\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$. **[2]**

(ii) Hence find the solution of the differential equation for which $y = 2$ when $x = 0$, giving your answer in the form $y = f(x)$. **[6]**

Q4, (Jan 2010, Q3)

Use the integrating factor method to find the solution of the differential equation

$$\frac{dy}{dx} + 2y = e^{-3x}$$

for which $y = 1$ when $x = 0$. Express your answer in the form $y = f(x)$. **[6]**

Q5, (Jan 2011, Q1)

(i) Find the general solution of the differential equation

$$\frac{dy}{dx} + xy = xe^{\frac{1}{2}x^2},$$

giving your answer in the form $y = f(x)$. **[4]**

(ii) Find the particular solution for which $y = 1$ when $x = 0$. **[2]**

Q6, (Jun 2011, Q3)

NOTE: This question requires methods used in Second-Order Differential Equations

The variables x and y satisfy the differential equation

$$\frac{dy}{dx} + 4y = 5\cos 3x.$$

(i) Find the complementary function. **[2]**

(ii) Hence, or otherwise, find the general solution. **[7]**

(iii) Find the approximate range of values of y when x is large and positive. **[2]**

Q7, (Jun 2012, Q3)

Find the solution of the differential equation

$$\frac{dy}{dx} + y\cot x = 2x$$

for which $y = 2$ when $x = \frac{1}{6}\pi$. Give your answer in the form $y = f(x)$. **[9]**

Q8, (Jan 2013, Q3)

Solve the differential equation $x\frac{dy}{dx} - 3y = x^4 e^{2x}$ for y in terms of x, given that $y = 0$ when $x = 1$. **[8]**

Q9, (Jun 2013, Q3)

The differential equation

$$3xy^2\frac{dy}{dx} + 2y^3 = \frac{\cos x}{x}$$

is to be solved for $x > 0$. Use the substitution $u = y^3$ to find the general solution for y in terms of x. **[8]**

Q10, (Jun 2014, Q2)

Use the substitution $u = y^2$ to find the general solution of the differential equation

$$\frac{dy}{dx} - 2y = \frac{e^x}{y}$$

for y in terms of x. **[8]**

Q11, (Jun 2015, Q5)

Find the particular solution of the differential equation

$$x\frac{dy}{dx} + 3y = x^2 + x$$

for which $y = 1$ when $x = 1$, giving y in terms of x. **[8]**

Q12, (Jun 2017, Q1)

Solve the differential equation

$$\frac{dy}{dx} + y\cot x = 9\operatorname{cosec} x$$

to find y in terms of x subject to the condition $y = \pi$ when $x = \frac{1}{6}\pi$. **[8]**
