Distance Between Planes and Lines (From OCR 4727)

Q1, (Jun 2007, Q6)

Lines l_1 and l_2 have equations

$$\frac{x-3}{2} = \frac{y-4}{-1} = \frac{z+1}{1} \quad \text{and} \quad \frac{x-5}{4} = \frac{y-1}{3} = \frac{z-1}{2}$$

respectively.

(i) Find the equation of the plane Π_1 which contains l_1 and is parallel to l_2, giving your answer in the form $\mathbf{r.n} = p$. [5]

(ii) Find the equation of the plane Π_2 which contains l_2 and is parallel to l_1, giving your answer in the form $\mathbf{r.n} = p$. [2]

(iii) Find the distance between the planes Π_1 and Π_2. [2]

(iv) State the relationship between the answer to part **(iii)** and the lines l_1 and l_2. [1]

Q2, (Jun 2010, Q1)

The line l_1 passes through the points (0, 0, 10) and (7, 0, 0) and the line l_2 passes through the points (4, 6, 0) and (3, 3, 1). Find the shortest distance between l_1 and l_2. **[7]**

Q3, (Jun 2010, Q7)

A line l has equation $\mathbf{r} = \begin{pmatrix} -7 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. A plane Π passes through the points (1, 3, 5) and (5, 2, 5), and is parallel to l.

(i) Find an equation of Π, giving your answer in the form $\mathbf{r.n} = p$. **[4]**

(ii) Find the distance between l and Π. **[4]**

(iii) Find an equation of the line which is the reflection of l in Π, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[4]**

Q4, (Jan 2011, Q2)

Two intersecting lines, lying in a plane p, have equations

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{-3} \quad \text{and} \quad \frac{x-1}{-1} = \frac{y-3}{2} = \frac{z-4}{4}.$$

(i) Obtain the equation of p in the form $2x - y + z = 3$. **[3]**

(ii) Plane q has equation $2x - y + z = 21$. Find the distance between p and q. **[3]**

Q5, (Jun 2011, Q1)

A line l has equation $\frac{x-1}{5} = \frac{y-6}{6} = \frac{z+3}{-7}$ and a plane p has equation $x + 2y - z = 40$.

(i) Find the acute angle between l and p. **[4]**

(ii) Find the perpendicular distance from the point (1, 6, −3) to p. **[2]**

Q6, (Jan 2012, Q4)

The line l has equations $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{2}$ and the point A is (7, 3, 7). M is the point where the perpendicular from A meets l.

(i) Find, in either order, the coordinates of M and the perpendicular distance from A to l. **[7]**

(ii) Find the coordinates of the point B on AM such that $\overrightarrow{AB} = 3\overrightarrow{BM}$. **[3]**

Q7, (Jan 2013, Q4)

The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix}1\\2\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\-1\end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix}3\\0\\1\end{pmatrix} + \mu\begin{pmatrix}4\\-1\\-1\end{pmatrix}$$

respectively.

(i) Find the shortest distance between the lines. **[5]**

(ii) Find a cartesian equation of the plane which contains l_1 and which is parallel to l_2. **[2]**

Q8, (Jun 2013, Q6)

The plane Π has equation $x + 2y - 2z = 5$. The line l has equation $\frac{x-1}{2} = \frac{y+1}{5} = \frac{z-2}{1}$.

(i) Find the coordinates of the point of intersection of l with the plane Π. **[3]**

(ii) Calculate the acute angle between l and Π. **[3]**

(iii) Find the coordinates of the two points on the line l such that the distance of each point from the plane Π is 2. **[5]**

Q9, (Jun 2015, Q6)

Find the shortest distance between the lines with equations

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1} \quad \text{and} \quad \frac{x-3}{4} = \frac{y-1}{-2} = \frac{z+1}{3}.$$ **[7]**

Q10, (Jun 2017, Q6)

The plane Π and the line l have equations

$$\mathbf{r}.\begin{pmatrix}2\\-3\\5\end{pmatrix} = 7 \text{ and } \mathbf{r} = \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$$

respectively. The point A has coordinates $(1, 2, -4)$.

(i) Find the shortest distance from the point A to the plane Π. **[3]**

(ii) Find the acute angle between Π and l. **[3]**

(iii) Find the point where the line parallel to l passing through A intersects the plane Π. **[4]**
