

**Complex Roots Of Equations and Using Exponential and Polar Form (From OCR 4727)**

**Q1, (Jun 2007, Q1)**

(i) By writing  $z$  in the form  $re^{i\theta}$ , show that  $zz^* = |z|^2$ . [1]

(ii) Given that  $zz^* = 9$ , describe the locus of  $z$ . [2]

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**Q2, (Jun 2008, Q7)**

The roots of the equation  $z^3 - 1 = 0$  are denoted by  $1, \omega$  and  $\omega^2$ .

(i) Sketch an Argand diagram to show these roots. [1]

(ii) Show that  $1 + \omega + \omega^2 = 0$ . [2]

(iii) Hence evaluate

(a)  $(2 + \omega)(2 + \omega^2)$ , [2]

(b)  $\frac{1}{2 + \omega} + \frac{1}{2 + \omega^2}$ . [2]

(iv) Hence find a cubic equation, with integer coefficients, which has roots  $2, \frac{1}{2 + \omega}$  and  $\frac{1}{2 + \omega^2}$ . [4]

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**Q3, (Jan 2009, Q2)**

(i) Express  $\frac{\sqrt{3} + i}{\sqrt{3} - i}$  in the form  $re^{i\theta}$ , where  $r > 0$  and  $0 \leq \theta < 2\pi$ . [3]

(ii) Hence find the smallest positive value of  $n$  for which  $\left(\frac{\sqrt{3} + i}{\sqrt{3} - i}\right)^n$  is real and positive. [2]

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**Q4, (Jun 2009, Q1)**

Find the cube roots of  $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$ , giving your answers in the form  $\cos \theta + i \sin \theta$ , where  $0 \leq \theta < 2\pi$ . [4]

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**Q5, (Jun 2010, Q3)**

In this question,  $w$  denotes the complex number  $\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi$ .

(i) Express  $w^2, w^3$  and  $w^*$  in polar form, with arguments in the interval  $0 \leq \theta < 2\pi$ . [4]

(ii) The points in an Argand diagram which represent the numbers

$$1, \quad 1 + w, \quad 1 + w + w^2, \quad 1 + w + w^2 + w^3, \quad 1 + w + w^2 + w^3 + w^4$$

are denoted by  $A, B, C, D, E$  respectively. Sketch the Argand diagram to show these points and join them in the order stated. (Your diagram need not be exactly to scale, but it should show the important features.) [4]

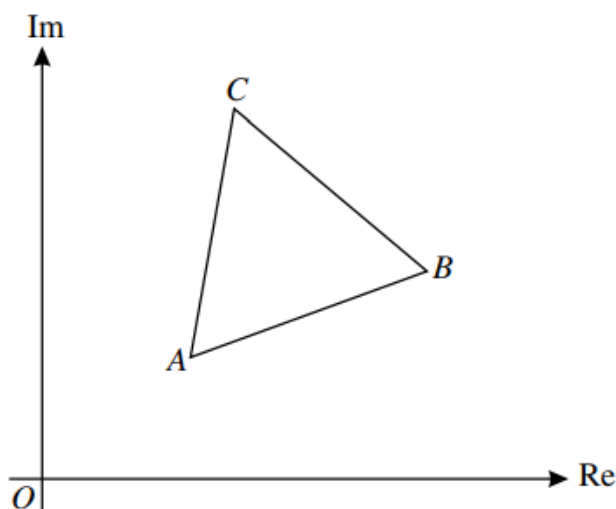
(iii) Write down a polynomial equation of degree 5 which is satisfied by  $w$ . [1]

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**Q6, (Jan 2011, Q4)**

The cube roots of 1 are denoted by 1,  $\omega$  and  $\omega^2$ , where the imaginary part of  $\omega$  is positive.

- (i) Show that  $1 + \omega + \omega^2 = 0$ . [2]



In the diagram,  $ABC$  is an equilateral triangle, labelled anticlockwise. The points  $A$ ,  $B$  and  $C$  represent the complex numbers  $z_1$ ,  $z_2$  and  $z_3$  respectively.

- (ii) State the geometrical effect of multiplication by  $\omega$  and hence explain why  $z_1 - z_3 = \omega(z_3 - z_2)$ . [4]
- (iii) Hence show that  $z_1 + \omega z_2 + \omega^2 z_3 = 0$ . [2]

**Q7, (Jan 2012, Q2)**

- (i) Show that  $(z^n - e^{i\theta})(z^n - e^{-i\theta}) \equiv z^{2n} - (2 \cos \theta) z^n + 1$ . [1]
- (ii) Express  $z^4 - z^2 + 1$  as the product of four factors of the form  $(z - e^{i\alpha})$ , where  $0 \leq \alpha < 2\pi$ . [6]

**Q8, (Jun 2012, Q2)**

- (i) Solve the equation  $z^4 = 2(1 + i\sqrt{3})$ , giving the roots exactly in the form  $r(\cos \theta + i \sin \theta)$ , where  $r > 0$  and  $0 \leq \theta < 2\pi$ . [5]
- (ii) Sketch an Argand diagram to show the lines from the origin to the point representing  $2(1 + i\sqrt{3})$  and from the origin to the points which represent the roots of the equation in part (i). [3]

**Q9, (Jun 2013, Q4)**

The complex numbers 0, 3 and  $3e^{\frac{1}{3}\pi i}$  are represented in an Argand diagram by the points  $O$ ,  $A$  and  $B$  respectively.

- (i) Sketch the triangle  $OAB$  and show that it is equilateral. [3]
- (ii) Hence express  $3 - 3e^{\frac{1}{3}\pi i}$  in polar form. [2]
- (iii) Hence find  $(3 - 3e^{\frac{1}{3}\pi i})^5$ , giving your answer in the form  $a + b\sqrt{3}i$  where  $a$  and  $b$  are rational numbers. [3]

**Q10, (Jun 2015, Q4)**

In an Argand diagram, the complex numbers  $0$ ,  $z$  and  $ze^{\frac{1}{6}i\pi}$  are represented by the points  $O$ ,  $A$  and  $B$  respectively.

- (i) Sketch a possible Argand diagram showing the triangle  $OAB$ . Show that the triangle is isosceles and state the size of angle  $AOB$ . [4]

The complex numbers  $1+i$  and  $5+2i$  are represented by the points  $C$  and  $D$  respectively. The complex number  $w$  is represented by the point  $E$ , such that  $CD = CE$  and angle  $DCE = \frac{1}{6}\pi$ .

- (ii) Calculate the possible values of  $w$ , giving your answers exactly in the form  $a+bi$ . [5]
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**Q11, (Jun 2016, Q1)**

In this question, give all non-real numbers in the form  $re^{i\theta}$  where  $r > 0$  and  $0 < \theta < 2\pi$ .

- (i) Solve  $z^5 = 1$ . [2]

- (ii) Hence, or otherwise, solve  $z^5 + 32 = 0$ . Sketch an Argand diagram showing the roots. [4]
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**Q12, (Jun 2017, Q5)**

In an Argand diagram the points  $O$ ,  $A$  and  $B$  are represented by the complex numbers  $0$ ,  $z$  and  $2e^{\frac{1}{3}i\pi}z$  respectively, where  $z$  is a complex number with modulus 5.

- (i) Calculate the exact area of the triangle  $OAB$ . [3]

The numbers  $-1+i$  and  $3+3i$  are represented by the points  $P$  and  $Q$  respectively. The complex number  $w$  is represented by the point  $R$ , such that  $PQ = PR$  and angle  $QPR = \frac{1}{4}\pi$ .

- (ii) Sketch an Argand diagram showing  $P$ ,  $Q$  and the two possible positions of  $R$ . Calculate the possible values of  $w$ , giving your answers in the form  $a+bi$ . [5]
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