Oblique Collisions (From OCR 4730)

Q1, (Jun 2006, Q5)

Two uniform smooth spheres $A$ and $B$, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision $A$ is moving with speed $12 \text{ m s}^{-1}$ at $60^\circ$ to the line of centres, and $B$ is moving with speed $8 \text{ m s}^{-1}$ along the line of centres (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [12]

Q2, (Jan 2007, Q6)

Two uniform smooth spheres $A$ and $B$ of equal radius are moving on a horizontal surface when they collide. $A$ has mass 0.4 kg, and $B$ has mass $m$ kg. Immediately before the collision, $A$ is moving with speed $4 \text{ m s}^{-1}$ at an acute angle $\theta$ to the line of centres, and $B$ is moving with speed $u \text{ m s}^{-1}$ at $30^\circ$ to the line of centres. Immediately after the collision $A$ is moving with speed $v \text{ m s}^{-1}$ at $45^\circ$ to the line of centres, and $B$ is moving with speed $3 \text{ m s}^{-1}$ perpendicular to the line of centres (see diagram).

(i) Find $u$. [2]

(ii) Given that $\theta = 88.1^\circ$ correct to 1 decimal place, calculate the approximate values of $v$ and $m$. [5]

(iii) The coefficient of restitution is 0.75. Show that the exact value of $\theta$ is a root of the equation $8 \sin \theta - 6 \cos \theta = 9 \cos 30^\circ$. [5]
Two uniform smooth spheres $A$ and $B$, of equal radius, have masses 4 kg and 3 kg respectively. They are moving on a horizontal surface, and they collide. Immediately before the collision, $A$ is moving with speed 15 m s$^{-1}$ at an angle $\alpha$ to the line of centres, where $\sin \alpha = 0.8$, and $B$ is moving along the line of centres with speed 12 m s$^{-1}$ (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [10]

Two smooth uniform spheres $A$ and $B$, of equal radius, have masses 3 kg and 4 kg respectively. They are moving on a horizontal surface, each with speed 5 m s$^{-1}$, when they collide. The directions of motion of $A$ and $B$ make angles $\alpha$ and $\beta$ respectively with the line of centres of the spheres, where $\sin \alpha = \cos \beta = 0.6$ (see diagram). The coefficient of restitution between the spheres is 0.75. Find the angle that the velocity of $A$ makes, immediately after impact, with the line of centres of the spheres. [10]

Two uniform smooth spheres $A$ and $B$, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, $A$ has speed 4 m s$^{-1}$ and is moving along the line of centres, and $B$ has speed $v$ m s$^{-1}$ and is moving perpendicular to the line of centres (see diagram). The coefficient of restitution is 0.6. The direction of motion of $B$ after the collision makes an angle of $45^\circ$ with the line of centres. Find the value of $v$. [7]
Q6, (Jun 2012, Q6)

Two smooth uniform spheres $A$ and $B$, of equal radius, have masses 2 kg and $m$ kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, $A$ has speed 5 m/s and is moving towards $B$ at an angle of $\alpha$ to the line of centres, where $\cos \alpha = 0.6$. $B$ has speed 2 m/s and is moving towards $A$ along the line of centres (see diagram). As a result of the collision, $A$’s loss of kinetic energy is 7.56 J, $B$’s direction of motion is reversed and $B$’s speed after the collision is 0.8 m/s. Find

(i) the speed of $A$ after the collision, [3]

(ii) the component of $A$’s velocity after the collision, parallel to the line of centres, stating with a reason whether its direction is to the left or to the right, [3]

(iii) the value of $m$, [3]

(iv) the coefficient of restitution between $A$ and $B$. [2]

Q7, (Jun 2014, Q3)

Two uniform smooth spheres $A$ and $B$ of equal radius are moving on a horizontal surface when they collide. $A$ has mass 0.1 kg and $B$ has mass 0.4 kg. Immediately before the collision $A$ is moving with speed 2.8 m/s along the line of centres, and $B$ is moving with speed 1 m/s at an angle $\theta$ to the line of centres, where $\cos \theta = 0.8$ (see diagram). Immediately after the collision $A$ is stationary. Find

(i) the coefficient of restitution between $A$ and $B$, [5]

(ii) the angle turned through by the direction of motion of $B$ as a result of the collision. [4]
Q8, (Jun 2015, Q5)

Two uniform smooth spheres $A$ and $B$, of equal radius, have masses $2\,\text{kg}$ and $m\,\text{kg}$ respectively. The spheres are moving on a horizontal surface when they collide. Before the collision, $A$ is moving with speed $a\,\text{m/s}$ in a direction making an angle $\alpha$ with the line of centres and $B$ is moving towards $A$ with speed $b\,\text{m/s}$ in a direction making an angle $\beta$ with the line of centres (see diagram). After the collision, $A$ moves with velocity $2\,\text{m/s}$ in a direction perpendicular to the line of centres and $B$ moves with velocity $2\,\text{m/s}$ in a direction making an angle of $45^\circ$ with the line of centres. The coefficient of restitution between $A$ and $B$ is $\frac{2}{3}$.

(i) Show that $a \cos \alpha = \frac{5}{6} \sqrt{2}$ and find $b \cos \beta$. [7]

(ii) Find the values of $a$ and $\alpha$. [4]

Q9, (Jun 2016, Q3)

Two uniform smooth spheres $A$ and $B$, of equal radius, have masses $2\,\text{kg}$ and $3\,\text{kg}$ respectively. The spheres are approaching each other on a horizontal surface when they collide. Before the collision $A$ is moving with speed $5\,\text{m/s}$ in a direction making an angle $\alpha$ with the line of centres, where $\cos \alpha = \frac{4}{5}$, and $B$ is moving with speed $3\frac{1}{4}\,\text{m/s}$ in a direction making an angle $\beta$ with the line of centres, where $\cos \beta = \frac{5}{13}$. A straight vertical wall is situated to the right of $B$, perpendicular to the line of centres (see diagram). The coefficient of restitution between $A$ and $B$ is $\frac{2}{3}$.

(i) Find the speed of $A$ after the collision. Find also the component of the velocity of $B$ along the line of centres after the collision. [7]

$B$ subsequently hits the wall.

(ii) Explain why $A$ and $B$ will have a second collision if the coefficient of restitution between $B$ and the wall is sufficiently large. Find the set of values of the coefficient of restitution for which this second collision will occur. [3]