Hooke's Law (From OCR 4730)

Q1, (Jun 2006, Q6)

(i)	T = 1470x/30	B1		
	[49x = 70x9.8]	M1		For using T = mg
	x = 14	A1		
	Distance fallen is 44m	A1ft	4	
(ii)	PE loss = 70g(30 + 14)	B1ft		
	EE gain = $1470 \times 14^2/(2 \times 30)$	B1ft		
	[$\frac{1}{2}$ $70v^2$ = 30184 – 4802]	M1		For a linear equation with terms representing KE, PE and EE changes.
	Speed is 26.9ms^{-1}	A1	4	AG
OR (ii)	[0.5 v^2 = 14g – 68.6 + 30g]	M1		For using Newton's 2nd law (vdv/dx = g – 0.7x), integrating (0.5 v^2 = gx – $0.35x^2$ + k), using $v(0)^2$ = 60g→ k = 30g, and substituting x = 14.
	For 14g + 30g	B1ft		
	For ∓68.6	B1ft		Accept in unsimplified form.
	Speed is 26.9ms^{-1}	A1	4	AG
(iii)	PE loss = 70g(30 + x)	B1ft		
	EE gain = $1470x^2/(2 \times 30)$	B1ft		
	[x^2 – 28x – 840 = 0]	M1		For using PE loss = KE gain to obtain a 3 term quadratic equation.
	Extension is 46.2m	A1	4	
OR (iii)		M1		For identifying SHM with n^2 = 1470/(70x30)
		M1		For using v_{max} = An
	$A = 26.9/\sqrt{0.7}$	A1		
	Extension is 46.2m	A1	4	

Q2, (Jan 2009, Q7)

(i)	Gain in EE = $20x^2/(2\text{x}2)$	B1	
	Loss in GPE = $0.8g(2 + x)$	B1	Accept $0.8gx$ if gain in KE is $\frac{1}{2}\,0.8(v^2 - 19.6)$
	[$\frac{1}{2}\,0.8v^2 = (15.68 + 7.84x) - 5x^2$]	M1	For using the p.c.energy
	$v^2 = 39.2 + 19.6x - 12.5x^2$	A1	AG
		[4]	
(ii)	**(a)**	M1	For attempting to solve $v^2 = 0$
	Maximum extension is 2.72m	A1	
		[2]	
	(b)		
	[$19.6 - 25x = 0$, $v^2 = 46.8832 - 12.5(x - 0.784)^2$]	M1	For solving $20x/2 = 0.8g$ or for differentiating and attempting to solve $d(v^2)/dx = 0$ or $dv/dx = 0$ or for expressing v^2 in the form $c - a(x - b)^2$.
	$x = 0.784$ or $c = 46.9$	A1	
	[$v_{max}{}^2 = 39.2 + 15.3664 - 7.6832$]	M1	For substituting $x = 0.784$ in the expression for v^2 or for evaluating $\sqrt{c}$
	Maximum speed is 6.85ms^{-1}	A1	
		[4]	
	(c)	M1	For using Newton's second law (3 terms required) or $a = v\,dv/dx$
	$\pm(0.8g - 20x/2) = 0.8a$ or $2v\,dv/dx = 19.6 - 25x$	A1	
	$a = \pm(9.8 - 12.5x)$ or $\ddot{y} = -12.5y$ where $y = x - 0.784$	A1	
	[$\lvert a\rvert_{max} = \lvert 9.8 - 12.5\text{x}2.72\rvert$ or $\lvert\ddot{y}_{max}\rvert = \lvert -12.5(2.72 - 0.784)\rvert$]	M1	For substituting x = ans(ii)(a) into $a(x)$ or y = ans(ii)(a) $- 0.784$ into $\ddot{y}(y)$
	Maximum magnitude is 24.2ms^{-2}	A1	
		[5]	

Q3, (Jan 2010, Q5)

(i)	Loss of EE = $120(0.5^2 - 0.3^2)/(2\times1.6)$ and gain in PE = 1.5×4	M1 A1	For using EE = $\lambda x^2/2L$ and PE = Wh
	v = 0 at B and loss of EE = gain in PE (= 6) ➔distance AB is 4m	M1 A1 [4]	For comparing EE loss and PE gain AG
(ii)	[120e/1.6 = 1.5] e = 0.02	M1 A1	For using T = mg and T = $\lambda x/L$
	Loss of EE = $120(0.5^2 - 0.02^2)/(2\times1.6)$ (or $120(0.3^2 - 0.02^2)/(2\times1.6)$)	B1ft	ft incorrect e only
	Gain in PE = 1.5(2.1- 1.6 – 0.02) (or 1.5(1.9 + 1.6 + 0.02) loss)	B1ft	ft incorrect e only
	[KE at max speed = 9.36 – 0.72 (or 3.36 + 5.28)]	M1	For using KE at max speed = Loss of EE – Gain (or + loss) in PE
	$\frac{1}{2}(1.5/9.8)v^2 = 9.36 - 0.72$ Maximum speed is 10.6 ms^{-1}	A1 A1 [7]	
	First alternative for **(ii)** x is distance AP $[\frac{1}{2}(1.5/9.8)v^2 + 1.5x + 120(0.5 - x)^2/3.2 = 120 \times 0.5^2/3.2]$	M1	For using energy at P = energy at A
	KE and PE terms correct EE terms correct $v^2 = 470.4x - 490x^2$	A1 A1 A1	
	[470.4 – 980x = 0] x = 0.48 Maximum speed is 10.6 ms^{-1}	M1 A1 A1	For attempting to solve $dv^2/dx = 0$
	Second alternative for **(ii)** [120e/1.6 = 1.5] e = 0.02	M1 A1	For using T = mg and T = $\lambda x/L$
	$[1.5 - 120(0.02 + x)/1.6 = 1.5\ddot{x}/g]$	M1	For using Newton's second law
	$n = \sqrt{490}$	M1 A1	For obtaining the equation in the form $\ddot{x} = -n^2x$, using $(AB - L - e_{equil})$ for amplitude and using v_{max} = na.
	a = 0.48 Maximum speed is 10.6 ms^{-1}	A1 A1	

Q4, (Jan 2012, Q4)

(i)	EE gain = $44.1x^2 \div (2x0.75)$ PE loss = $1.8g(0.75 + x)$ $[x^2 - 0.6x - 0.45 = 0]$ Extension is 1.03 m	B1 B1 M1 A1 **[4]**	 ignore signs For using EE gain = PE loss	allow use of $(e + x)$ for x $44.1x^2$-$26.46x$-19.845=0 allow sign errors 1.0348469...
(ii)	$\frac{44.1 \times 1.03}{0.75} - 1.8 \times 9.8 = -1.8\ddot{x}$ Acceleration is $-24.0\ \text{ms}^{-2}$	M1 M1 A1ft A1 **[4]**	For using $T = \lambda x/L$ For using Newton's 2nd law ft their '1.03' from (i) direction must be clear	 allow missed g, m, sign error allow sign error 1.03 → − 23.84666 1.035 → − 24.01

Q5, (Jun 2007, Q6)

(i) [T = 2058x/5.25]	M1		For using T = λ x/L
2058x/5.25 = 80 x 9.8 (x = 2)	A1		
OP = 7.25m	A1	3	AG From 5.25 + 2
(ii) Initial PE = (80 + 80)g(5) (= 7840) or (80 + 80)gX used in energy equation	B1		
Initial KE = ½ (80 + 80)3.5^2 (= 980)	B1		
[Initial EE = $2058x2^2/(2x5.25)$ (= 784), Final EE = $2058x7^2/(2x5.25)$ (= 9604), or $2058(X + 2)^2/(2x5.25)$]	M1		For using EE = $\lambda x^2/2L$
[Initial energy = 7840 + 980 + 784, final energy = 9604 or 1568X + 980 + 784 = $196(X^2 + 4X + 4)$ → $196X^2 - 784X - 980 = 0$]	M1		For attempting to verify compatibility with the principle of conservation of energy, or using the principle and solving for X
Initial energy = final energy or X = 5 →P&Q just reach the net	A1	5	AG
(iii) [PE gain = 80g(7.25 + 5)]	M1		For finding PE gain from net level to O
PE gain = 9604	A1		
PE gain = EE at net level → P just reaches O	A1	3	AG
(iv) For any one of 'light rope', 'no air resistance', 'no energy lost in rope'	B1		
For any other of the above	B1	2	

FIRST ALTERNATIVE METHOD FOR PART (ii)

[160g – 2058x/5.25 = 160v dv/dx]	M1		For using Newton's second law with a = v dv/dx, separating the variables and attempting to integrate
$v^2/2 = gx - 1.225x^2$ (+ C)	A1		Any correct form
	M1		For using v(2) = 3.5
C = -8.575	A1		
$[v(7)^2]/2$ = 68.6 – 60.025 – 8.575 = 0 →P&Q just reach the net	A1	5	AG

SECOND ALTERNATIVE METHOD FOR PART (ii)

$\ddot{x} = g - 2.45x$ (= -2.45(x – 4))	B1		
	M1		For using $n^2 = 2.45$ and $v^2 = n^2(A^2 - (x-4)^2)$
$3.5^2 = 2.45(A^2 - (-2)^2)$ (A = 3)	A1		
[(4 – 2) + 3]	M1		For using 'distance travelled downwards by P and Q = distance to new equilibrium position + A
distance travelled downwards by P and Q = 5 →P&Q just reach the net	A1	5	AG

Q6, (Jun 2012, Q7)

(i)	$E_{(AP=2.9)} = 120 \times 0.9^2/4 + 180 \times 0.1^2/6$ $= (24.3 + 0.3)$ and $E_{(AP=2.1)} = 120 \times 0.1^2/4 + 180 \times 0.9^2/6$ $= (0.3 + 24.3)$ ➔same for each position Conservation of energy ➔ $v = 0$ when AP $= 2.1$, string taut here so taut throughout motion – oe,	M1 A1 B1 **[3]**	For using EPE = $\lambda x^2/2L$ for both strings for one position 24.6 seen twice Need to point out that $v = 0$ when $AP = 2.1$ or KE = 0 Dep on M1A1
(ii)	$T_A = 120(0.5 + x)/2$, $T_B = 180(0.5 - x)/3$ $[(30 - 60x) - (30 + 60x) = (+/-)0.8a]$ $a = -150x$	B1 M1 A1 **[3]**	soi For using Newton's 2nd law; allow omission of 0.8 With no wrong working
(iii)	SHM because $a = -k$ (where $k > 0$) $[T = 2\pi/\sqrt{150}]$ Time interval is 0.257 s	M1 M1 A1 FT **[3]**	SHM because $a = -\omega^2 x$ or in words For using $T = 2\pi/n$; must follow from (ii) FT π ÷candidate's n 0.256509…
(iv)	$[x = 0.4\cos(\sqrt{150} \times 0.6) = 0.194]$ [distance $= 4a + (a - 0.194)$] Distance travelled is 1.81 m	M1 M1 A1 **[3]**	For using $x = a\cos(0.6n)$, where n follows from (ii) and a is numerical. For using $T < 0.6 < 1.25T$ ➔ distance $= 4a + (a - x)$; may be implied by $1.6 <$ distance < 2.0 CAO, no wrong working
(v)	 Speed is 4.29 ms^{-1}.	M1 A1 **[2]**	For using $\dot{x} = -an\sin(0.6n)$, where n follows from (ii) Or using $v^2 = n^2(a^2 - x^2)$, where n follows from (ii) and x follows from (iv) or using $\dot{x} = an\cos(0.6n)$ if $x = a\sin(0.6n)$ used in (iv), where n follows from (ii) Condone −4.29

Q7, (Jun 2013, Q1)

Use of $T = \frac{\lambda e}{l}$	M1	Attempt at one tension; allow use of x	allow $2l$ for M1
	A1	$\frac{20(d-0.4)}{0.4}$ or $\frac{30(d-0.6)}{0.6}$	either term seen, accept in terms of x
Weight = tension 1 + tension 2	M1		condone Wg and W/g
	A1	$100 = 50d - 20 + 50d - 30$	fractions and brackets removed
($AW =$) 1.5 (m)	A1		
	[5]		

Q8, (Jun 2014, Q2)

(i)	By energy	M1*	Attempt at elastic energy	Allow M1 for $\frac{30y^2}{(2)\times 0.6} = kd$
	$\frac{30(d-0.6)^2}{2\times 0.6} = 48 \times d$	A1		$\frac{30x^2}{2\times 0.6} = 48(x+0.6)$
	$25d^2 - 78d + 9 = 0$ or $30d^2 - 93.6d + 10.8 = 0$	*M1	get 3 term quadratic and attempt to solve	allow 1 slip or $25x^2 - 48x - 28.8 = 0$
	($d =$) 3 (m)	A1 **[4]**	ignore $d = 0.12$, unless given as answer	($x =$) 2.4 leading to ($d =$) 3
(ii)	Use $F = ma$	M1		allow missing g, allow 1.3 or 0.6 to be omitted
	$48 - \frac{30\times(3-0.6-1.3)}{0.6} = (\pm)\frac{48}{g}a$	A1ft	ft their '3'	Using energy:
	($a =$) (+/–) 1.43	A1	1.4291666	$a = v\frac{dv}{dx} = \frac{g}{48}(50x - 72)$ M1A1
	upwards	A1 **[4]**	depends on a being right	