

Centres of Mass Involving Integration (From OCR 4763)

Q1, (Jan 2006, Q4)

(i)	$\int y \, dx = \int_0^2 (4 - x^2) \, dx = \left[4x - \frac{1}{3}x^3 \right]_0^2 \quad (= \frac{16}{3})$ $\int xy \, dx = \int_0^2 x(4 - x^2) \, dx$ $= \left[2x^2 - \frac{1}{4}x^4 \right]_0^2 \quad (= 4)$ $\bar{x} = \frac{4}{\frac{16}{3}}$ $= 0.75$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>E1</p>	<p>Correctly obtained</p>
	$\int \frac{1}{2}y^2 \, dx = \int_0^2 \frac{1}{2}(16 - 8x^2 + x^4) \, dx$ $= \left[8x - \frac{4}{3}x^3 + \frac{1}{10}x^5 \right]_0^2 \quad (= \frac{128}{15})$	<p>M1</p> <p>A1</p>	
	<p>OR $\int yx \, dy = \int_0^4 y\sqrt{4-y} \, dy$</p> $= \left[-\frac{2}{3}y(4-y)^{\frac{3}{2}} - \frac{4}{15}(4-y)^{\frac{5}{2}} \right]_0^4$	<p>M1</p> <p>A1</p>	<p>Valid method of integration</p> <p>or $\left[-\frac{8}{3}(4-y)^{\frac{3}{2}} + \frac{2}{5}(4-y)^{\frac{5}{2}} \right]_0^4$</p>
	$\bar{y} = \frac{\frac{128}{15}}{\frac{16}{3}}$ $= 1.6$	<p>M1</p> <p>E1</p>	<p>Correctly obtained</p> <p>9 SR If $\frac{1}{2}$ is omitted, marks for \bar{y} are M1A0M0E0</p>
(ii)	$\bar{x} = \frac{12 \times 0 + 6.5 \times 0.75 + 6.5 \times 2.75}{12 + 6.5 + 6.5}$ $= \frac{22.75}{25} = 0.91$ $\bar{y} = \frac{12 \times 0 + 6.5 \times 1.6 + 6.5 \times 1.6}{25}$ $= \frac{20.8}{25} = 0.832$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For $6.5 \times 0.75 + 6.5 \times 2.75$</p> <p>Using $(\sum m)\bar{x} = \sum mx$</p> <p>Using $(\sum m)\bar{y} = \sum my$</p> <p>5</p>
(iii)	$\tan \theta = \frac{2 - 0.91}{4 - 0.832} \quad (= \frac{1.09}{3.168})$ $\theta = 19.0^\circ$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>For CM vertically below A</p> <p>For trig in a triangle containing θ, or finding the gradient of AG</p> <p>Correct expression for $\tan \theta$ or $\tan(90 - \theta)$</p> <p>Accept 0.33 rad</p> <p>4</p>

Q2, (June 2006, Q4)

(i)	$\int \pi y^2 dx = \int_1^4 \pi x dx$ $= \left[\frac{1}{2} \pi x^2 \right]_1^4 = 7.5\pi$ $\int \pi x y^2 dx$ $= \int_1^4 \pi x^2 dx = \left[\frac{1}{3} \pi x^3 \right]_1^4 (= 21\pi)$ $\bar{x} = \frac{21\pi}{7.5\pi}$ $= 2.8$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>6</p>	<p>π may be omitted throughout</p>
(ii)	<p>Cylinder has mass $3\pi\rho$ Cylinder has CM at $x = 2.5$</p> $(4.5\pi\rho)\bar{x} + (3\pi\rho)(2.5) = (7.5\pi\rho)(2.8)$ $\bar{x} = 3$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>5</p>	<p>Or volume 3π</p> <p>Relating three CMs <i>(ρ and / or π may be omitted)</i> or equivalent, e.g.</p> $\bar{x} = \frac{(7.5\pi\rho)(2.8) - (3\pi\rho)(2.5)}{7.5\pi\rho - 3\pi\rho}$ <p>Correctly obtained</p>
(iii)(A)	<p>Moments about A, $S \times 3 - 96 \times 2 = 0$ $S = 64 \text{ N}$</p> <p>Vertically, $R + S = 96$</p> $R = 32 \text{ N}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>4</p>	<p>Moments equation</p> <p>or another moments equation <i>Dependent on previous M1</i></p>
(B)	<p>Moments about A, $S \times 3 - 96 \times 2 - 6 \times 1.5 = 0$</p> <p>Vertically, $R + S = 96 + 6$ $R = 35 \text{ N}, S = 67 \text{ N}$</p> <hr/> <p>OR Add 3 N to each of R and S $R = 35 \text{ N}, S = 67 \text{ N}$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>3</p>	<p>Moments equation</p> <p>Both correct</p> <p><i>Provided $R \neq S$</i> Both correct</p>

Q3, (Jan 2007, Q4)

(i)	<p>Area is $\int_1^a \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_1^a$ $= 1 - \frac{1}{a}$</p> <p>$\int xy dx = \int_1^a \frac{1}{x} dx = (\ln a)$</p> <p>$\bar{x} = \frac{\int xy dx}{\int y dx}$ $= \frac{\ln a}{1 - \frac{1}{a}} \quad \left(= \frac{a \ln a}{a - 1} \right)$</p> <p>$\int \frac{1}{2} y^2 dx = \int_1^a \frac{1}{2x^4} dx = \left[-\frac{1}{6x^3} \right]_1^a$ $= \frac{1}{6} \left(1 - \frac{1}{a^3} \right)$</p> <p>$\bar{y} = \frac{\int \frac{1}{2} y^2 dx}{\int y dx}$ $= \frac{\frac{1}{6} \left(1 - \frac{1}{a^3} \right)}{1 - \frac{1}{a}} = \frac{a^3 - 1}{6(a^3 - a^2)}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>E1</p>	<p>Condone omission of $\frac{1}{2}$</p> <p>($\frac{1}{2}$ needed for this mark)</p> <p>8</p>
(ii)	<p>When $a = 2$, $\bar{x} = 2 \ln 2$, $\bar{y} = \frac{7}{24}$</p> <p>$\tan \theta = \frac{\bar{x} - 1}{1 - \bar{y}}$ $= \frac{2 \ln 2 - 1}{1 - \frac{7}{24}}$ $\theta = 28.6^\circ$</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>CM vertically below A</p> <p>Correct expression for $\tan \theta$ or $\tan(90 - \theta)$</p> <p>3</p>

Q4, (Jun 2007, Q4)

<p>(a)</p>	$\text{Area is } \int_0^2 x^3 dx = \left[\frac{1}{4}x^4 \right]_0^2 = 4$ $\int xy dx = \int_0^2 x^4 dx$ $= \left[\frac{1}{5}x^5 \right]_0^2 = 6.4$ $\bar{x} = \frac{6.4}{4} = 1.6$ $\int \frac{1}{2}y^2 dx = \int_0^2 \frac{1}{2}x^6 dx$ $= \left[\frac{1}{14}x^7 \right]_0^2 = \frac{64}{7}$ $\bar{y} = \frac{\int \frac{1}{2}y^2 dx}{\int y dx}$ $= \frac{\frac{64}{7}}{4} = \frac{16}{7}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p></p> <p>Condone omission of $\frac{1}{2}$</p> <p>Accept 2.3 from correct working</p> <p>8</p>
<p>(b)(i)</p>	$\text{Volume is } \int \pi y^2 dx = \int_1^2 \pi(4 - x^2) dx$ $= \pi \left[4x - \frac{1}{3}x^3 \right]_1^2 = \frac{5}{3}\pi$ $\int \pi x y^2 dx = \int_1^2 \pi x(4 - x^2) dx$ $= \pi \left[2x^2 - \frac{1}{4}x^4 \right]_1^2 = \frac{9}{4}\pi$ $\bar{x} = \frac{\int \pi x y^2 dx}{\int \pi y^2 dx}$ $= \frac{\frac{9}{4}\pi}{\frac{5}{3}\pi} = \frac{27}{20} = 1.35$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>E1</p>	<p>π may be omitted throughout</p> <p>For $\frac{5}{3}$</p> <p>For $\frac{9}{4}$</p> <p>Must be fully correct</p> <p>6</p>
<p>(ii)</p>	<p>Height of solid is $h = 2\sqrt{3}$</p> <p>$T h = mg \times 0.35$</p> <p>$F = T = 0.101mg, R = mg$</p> <p>Least coefficient of friction is $\frac{F}{R} = 0.101$</p>	<p>B1</p> <p>M1</p> <p>F1</p> <p>A1</p>	<p>Taking moments</p> <p>Must be fully correct</p> <p>4 (e.g. A0 if $m = \frac{5}{3}\pi$ is used)</p>

Q5, (Jan 2008, Q4)

<p>(i)</p>	$V = \int_1^8 \pi \left(x^{-\frac{1}{3}}\right)^2 dx$ $= \pi \left[3x^{\frac{1}{3}} \right]_1^8 = 3\pi$ $V\bar{x} = \int_1^8 \pi x \left(x^{-\frac{1}{3}}\right)^2 dx$ $= \pi \left[\frac{3}{4} x^{\frac{4}{3}} \right]_1^8 = \frac{45}{4}\pi$ $\bar{x} = \frac{\frac{45}{4}\pi}{3\pi}$ $= \frac{15}{4} = 3.75$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>6</p>	<p>π may be omitted throughout</p> <p>Dependent on previous M1M1</p>
<p>(ii)</p>	$A = \int_1^8 x^{-\frac{1}{3}} dx$ $= \left[\frac{3}{2} x^{\frac{2}{3}} \right]_1^8 = \frac{9}{2} = 4.5$ $A\bar{x} = \int_1^8 x \left(x^{-\frac{1}{3}}\right) dx$ $= \left[\frac{3}{5} x^{\frac{5}{3}} \right]_1^8 = \frac{93}{5} = 18.6$ $\bar{x} = \frac{18.6}{4.5} = \frac{62}{15} (\approx 4.13)$ $A\bar{y} = \int_1^8 \frac{1}{2} \left(x^{-\frac{1}{3}}\right)^2 dx$ $= \left[\frac{3}{2} x^{\frac{1}{3}} \right]_1^8 = \frac{3}{2} = 1.5$ $\bar{y} = \frac{1.5}{4.5} = \frac{1}{3}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>8</p>	<p>If $\frac{1}{2}$ omitted, award M1A0A0</p>

Q6, (Jan 2009, Q4)

<p>(a)</p>	$\int \frac{1}{2} y^2 dx = \int_{-a}^a \frac{1}{2} (a^2 - x^2) dx$ $= \left[\frac{1}{2} (a^2 x - \frac{1}{3} x^3) \right]_{-a}^a$ $= \frac{2}{3} a^3$ $\bar{y} = \frac{\frac{2}{3} a^3}{\frac{1}{2} \pi a^2}$ $= \frac{4a}{3\pi}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>E1</p>	<p>For integral of $(a^2 - x^2)$</p> <p><i>Dependent on previous MI</i></p> <p>4</p>
<p>(b)(i)</p>	$V = \int \pi y^2 dx = \int_0^h \pi (mx)^2 dx$ $= \left[\frac{1}{3} \pi m^2 x^3 \right]_0^h = \frac{1}{3} \pi m^2 h^3$ $\int \pi xy^2 dx = \int_0^h \pi x (mx)^2 dx$ $= \left[\frac{1}{4} \pi m^2 x^4 \right]_0^h = \frac{1}{4} \pi m^2 h^4$ $\bar{x} = \frac{\frac{1}{4} \pi m^2 h^4}{\frac{1}{3} \pi m^2 h^3}$ $= \frac{3}{4} h$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>E1</p>	<p>π may be omitted throughout</p> <p>For integral of x^2 or use of $V = \frac{1}{3} \pi r^2 h$ and $r = mh$</p> <p>For integral of x^3</p> <p><i>Dependent on MI for integral of x^3</i></p> <p>6</p>
<p>(ii)</p>	$m_1 = \frac{1}{3} \pi \times 0.7^2 \times 2.4 \rho = \frac{1}{3} \pi \rho \times 1.176$ $VG_1 = 1.8$ $m_2 = \frac{1}{3} \pi \times 0.4^2 \times 1.1 \rho = \frac{1}{3} \pi \rho \times 0.176$ $VG_2 = 1.3 + \frac{3}{4} \times 1.1 = 2.125$ $(m_1 - m_2)(VG) + m_2(VG_2) = m_1(VG_1)$ $(VG) + 0.176 \times 2.125 = 1.176 \times 1.8$ <p>Distance (VG) is 1.74 m</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>F1</p> <p>A1</p>	<p>For m_1 and m_2 (or volumes) or $\frac{1}{4} \times 1.1$ from base</p> <p>Attempt formula for composite body</p> <p>5</p>
<p>(iii)</p>	<p>VQG is a right-angle</p> $VQ = VG \cos \theta \text{ where } \tan \theta = \frac{0.7}{2.4} \quad (\theta = 16.26^\circ)$ $VQ = 1.7428 \times \frac{24}{25}$ $= 1.67 \text{ m}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>ft is $VG \times 0.96$</p> <p>3</p>

Q7, (Jun 2010, Q3)

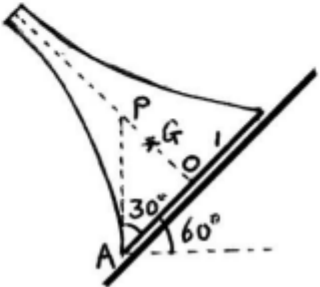
<p>(i)</p>	<p>Volume is $\int_1^5 \pi \left(\frac{1}{x}\right)^2 dx$ $= \pi \left[-\frac{1}{x} \right]_1^5 \quad (= \frac{4}{5} \pi)$</p> <p>$\int \pi x y^2 dx = \int_1^5 \pi x \left(\frac{1}{x}\right)^2 dx$ $= \pi \left[\ln x \right]_1^5 \quad (= \pi \ln 5)$</p> <p>$\bar{x} = \frac{\pi \ln 5}{\frac{4}{5} \pi} = \frac{5 \ln 5}{4} \quad (2.012)$</p>	<p>M1 A1 M1 A1 A1</p>	<p>π may be omitted throughout <i>Limits not required</i></p> <p>For $-\frac{1}{x}$</p> <p><i>Limits not required</i></p> <p>For $\ln x$</p> <p>SR If exact answers are not seen, deduct only the first A1 affected</p> <p>5</p>
<p>(ii)</p>	<p>Area is $\int_1^5 \frac{1}{x} dx$ $= \left[\ln x \right]_1^5 \quad (= \ln 5)$</p> <p>$\int x y dx = \int_1^5 x \left(\frac{1}{x}\right) dx \quad (= \left[x \right]_1^5 = 4)$</p> <p>$\bar{x} = \frac{4}{\ln 5} \quad (2.485)$</p> <p>$\int \frac{1}{2} y^2 dx = \int_1^5 \frac{1}{2} \left(\frac{1}{x}\right)^2 dx$ $= \left[-\frac{1}{2x} \right]_1^5 \quad (= \frac{2}{5})$</p> <p>$\bar{y} = \frac{\frac{2}{5}}{\ln 5} = \frac{2}{5 \ln 5} \quad (0.2485)$</p>	<p>M1 A1 M1 A1 M1 A1 A1</p>	<p><i>Limits not required</i></p> <p>For $\ln x$</p> <p><i>Limits not required</i></p> <p>For $\int \left(\frac{1}{x}\right)^2 dx$</p> <p>For $-\frac{1}{2x}$</p> <p>7</p>
<p>(iii)</p>	<p>CM of R_2 is $\left(\frac{2}{5 \ln 5}, \frac{4}{\ln 5} \right)$</p>	<p>B1B1 ft</p>	<p><i>Do not penalise inexact answers in this part</i></p> <p>2</p>
<p>(iv)</p>	<p>$\bar{x} = \frac{(\ln 5) \left(\frac{4}{\ln 5}\right) + (\ln 5) \left(\frac{2}{5 \ln 5}\right) + (1) \left(\frac{1}{2}\right)}{\ln 5 + \ln 5 + 1}$</p> <p>CM is $\left(\frac{4.9}{2 \ln 5 + 1}, \frac{4.9}{2 \ln 5 + 1} \right) \quad (1.161, 1.161)$</p>	<p>B1 M1 M1 A1 cao</p>	<p>For CM of R_3 is $\left(\frac{1}{2}, \frac{1}{2}\right)$ (one coordinate is sufficient)</p> <p>Using $\sum mx$ with three terms</p> <p>Using $\frac{\sum mx}{\sum m}$ with at least two terms in each sum</p> <p>4</p>

Q8, (Jun 2013, Q4)

(a)	(i)	$V = \int_0^4 \pi x^2 (4-x) dx$ $= \pi \left[\frac{4}{3} x^3 - \frac{1}{4} x^4 \right]_0^4 \quad (= \frac{64\pi}{3})$ $V\bar{x} = \int \pi xy^2 dx = \int_0^4 \pi x^3 (4-x) dx$ $= \pi \left[x^4 - \frac{1}{5} x^5 \right]_0^4 \quad (= 51.2\pi)$ $\bar{x} = \frac{51.2\pi}{\frac{64}{3}\pi}$ $= 2.4$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>For $\int (x\sqrt{4-x})^2 dx$</p> <p>For $\frac{4}{3}x^3 - \frac{1}{4}x^4$</p> <p>For $\int xy^2 dx$</p> <p>For $x^4 - \frac{1}{5}x^5$</p> <p><i>Dependent on previous M1M1</i></p>	<p>π may be omitted throughout</p>	
	(a)	(ii)	$W(2.4 \sin \theta) = W(4 \cos \theta)$ $\tan \theta = \frac{4}{2.4} = \frac{5}{3}$ $\theta = 59.0^\circ \quad (3 \text{ sf})$	<p>M1</p> <p>A1</p>	<p>Taking moments</p> <p>FT Correct equation for required angle</p>	<p>$W(2.4 \cos \phi) = W(4 \sin \phi)$ is A0 unless $\theta = 90^\circ - \phi$ also appears</p>
				<p>A1</p> <p>[3]</p>	<p>FT is $\tan^{-1} \frac{4}{\bar{x}}$</p>	<p>FT requires $\bar{x} < 4$</p>

(b)	$x = 2 + y^{\frac{1}{3}}$ $A = \int_0^8 (2 + y^{\frac{1}{3}}) dy = \left[2y + \frac{3}{4}y^{\frac{4}{3}} \right]_0^8 \quad (= 28)$ $A\bar{x} = \int \frac{1}{2}x^2 dy = \int_0^8 \frac{1}{2} \left(4 + 4y^{\frac{1}{3}} + y^{\frac{2}{3}} \right) dy$ $= \left[2y + \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{10}y^{\frac{5}{3}} \right]_0^8 \quad (= 49.6)$ $\bar{x} = \frac{49.6}{28} = \frac{62}{35} = 1.77 \quad (3 \text{ sf})$ $A\bar{y} = \int xy dy = \int_0^8 \left(2y + y^{\frac{4}{3}} \right) dy$ $= \left[y^2 + \frac{3}{7}y^{\frac{7}{3}} \right]_0^8 \quad \left(= \frac{832}{7} \right)$ $\bar{y} = \frac{\frac{832}{7}}{28} = \frac{208}{49} = 4.24 \quad (3 \text{ sf})$	<p>B1</p> <p>B1</p> <p>M1</p> <p>B2</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>FT</p> <p>For $\int x^2 dy$</p> <p>FT for $2y + \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{10}y^{\frac{5}{3}}$</p> <p>Give B1 for one minor slip in integration, or if $\frac{1}{2}$ omitted</p> <p>CAO</p> <p>For $\int xy dy$</p> <p>FT for $y^2 + \frac{3}{7}y^{\frac{7}{3}}$</p> <p>CAO</p>	<p>Or $32 - \left[\frac{1}{4}(x-2)^4 \right]_2^4$</p> <p>Or $32 \times 2 - \int_2^4 xy dx$</p> <p>Or $\frac{1}{5}(x-2)^5 + \frac{1}{2}(x-2)^4$</p> <p>Or $\frac{1}{4}x(x-2)^4 - \frac{1}{20}(x-2)^5$</p> <p>Or $\frac{1}{5}x^5 - \frac{3}{2}x^4 + 4x^3 - 4x^2$</p> <p><i>Must be \bar{x}</i></p> <p>Or $32 \times 4 - \int_2^4 \left(\frac{1}{2}\right)y^2 dx$</p> <p>Or B2 for $\frac{1}{14}(x-2)^7$</p> <p>Give B1 for one minor slip in integration, or if $\frac{1}{2}$ omitted</p> <p><i>Must be \bar{y}</i></p>
OR	<p>Region under curve has CM $(3.6, \frac{16}{7})$</p> $28\bar{x} + 4 \times 3.6 = 32 \times 2$ $\bar{x} = 1.77$ $28\bar{y} + 4 \times \frac{16}{7} = 32 \times 4$ $\bar{y} = 4.24$	<p>[9]</p>	<p>B2B2</p> <p>B1 (for 28)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For integrals, as above</p>

Q9, (Jun 2014, Q4)

<p>(i)</p>	<p>Volume is $\int_0^k \pi(e^{-x})^2 dx$</p> $= \pi \left[-\frac{1}{2}e^{-2x} \right]_0^k \quad \{ = \frac{1}{2}\pi(1 - e^{-2k}) \}$ <p>$\int \pi xy^2 dx$</p> $= \int_0^k \pi x e^{-2x} dx = \pi \left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right]_0^k$ $= \frac{1}{4}\pi(1 - 2ke^{-2k} - e^{-2k})$ $\bar{x} = \frac{1 - 2ke^{-2k} - e^{-2k}}{2(1 - e^{-2k})}$ $= \frac{1 - e^{-2k}}{2(1 - e^{-2k})} - \frac{2ke^{-2k}}{2(1 - e^{-2k})} = \frac{1}{2} - \frac{k}{e^{2k} - 1}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1A1</p> <p>A1</p> <p>E1</p> <p>[7]</p>	<p>π may be omitted throughout</p> <p>For $-\frac{1}{2}e^{-2x}$</p> <p>For $-\frac{1}{2}xe^{-2x}$ and $-\frac{1}{4}e^{-2x}$</p> <p>Any correct form</p>	
<p>(ii)</p>	<p>$OG < \frac{1}{2}$ for all values of k</p> <p>$OP = (1) \tan 30^\circ = \frac{1}{\sqrt{3}} (= 0.577)$</p> <p>$OG < OP$ (or $\hat{OAG} < 30^\circ$) so G is to the right of AP and solid will not topple</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>E1</p> <p>[4]</p>	<p>OR $\frac{k}{e^{2k} - 1} > 0$ o.e. stated or implied</p> <p>Allow $\bar{x} \rightarrow \frac{1}{2}$ as $k \rightarrow \infty$ for B1</p> <p>Trigonometry in OAP or OAG</p> <p>Or $\hat{OAG} < \tan^{-1} \frac{1}{2} (= 26.6^\circ)$</p> <p>Fully correct explanation</p>	

(iii)	<p>Area is $\int_0^k e^{-x} dx = [-e^{-x}]_0^k (=1-e^{-k})$</p> <p>$\int xy dx$</p> <p>$= \int_0^k xe^{-x} dx = [-xe^{-x} - e^{-x}]_0^k$</p> <p>$\bar{x} = \frac{1-ke^{-k} - e^{-k}}{1-e^{-k}}$</p> <p>$\int \frac{1}{2}y^2 dx$</p> <p>$= \int_0^k \frac{1}{2}e^{-2x} dx = [-\frac{1}{4}e^{-2x}]_0^k$</p> <p>$\bar{y} = \frac{1-e^{-2k}}{4(1-e^{-k})}$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>Any correct form</p> <p>For $\int \dots y^2 dx$</p> <p>Any correct form</p>	<p>e.g. $1 - \frac{k}{e^k - 1}$</p> <p>e.g. $\frac{1}{4}(1 + e^{-k})$</p>
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(Q10, Jun 2015, Q4)

(a)	Area is $\int_0^a \frac{x^2(a-x)}{a^2} dx$	M1	
	$= \left[\frac{x^3}{3a} - \frac{x^4}{4a^2} \right]_0^a \quad (= \frac{a^2}{12})$	A1	
	$\int xy dx$	M1	
	$= \int_0^a \frac{x^3(a-x)}{a^2} dx = \left[\frac{x^4}{4a} - \frac{x^5}{5a^2} \right]_0^a \quad (= \frac{a^3}{20})$	A1	
	$\bar{x} = \frac{\frac{1}{20}a^3}{\frac{1}{12}a^2} = \frac{3a}{5}$	A1	
	$\int \frac{1}{2} y^2 dx = \int_0^a \frac{x^4(a-x)^2}{2a^4} dx$	M1	For $\int \dots y^2 dx$
	$= \left[\frac{x^5}{10a^2} - \frac{x^6}{6a^3} + \frac{x^7}{14a^4} \right]_0^a \quad (= \frac{a^3}{210})$	A2	Give A1 if just one error (e.g. omission of factor $\frac{1}{2}$)
	$\bar{y} = \frac{\frac{1}{210}a^3}{\frac{1}{12}a^2} = \frac{2a}{35} \quad (\approx 0.0571a)$	A1	
	[9]		

(b)	(i)	<p>Volume is $\int_0^3 \pi(x^2 + 16) dx$</p> $= \pi \left[\frac{x^3}{3} + 16x \right]_0^3 \quad (= 57\pi)$ <p>$\int \pi xy^2 dx$</p> $= \int_0^3 \pi x(x^2 + 16) dx = \pi \left[\frac{x^4}{4} + 8x^2 \right]_0^3 \quad (= \frac{369}{4}\pi)$ $\bar{x} = \frac{\frac{369}{4}\pi}{57\pi} = \frac{123}{76} \quad (\approx 1.62)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	π may be omitted throughout	<i>Condone consistent use of $2\pi y^2$ etc</i>
(b)	(ii)	<p>Volume of A and B combined is $\pi \times 5^2 \times 3 = 75\pi$</p> $(18\pi)\bar{x}_B + (57\pi)\left(\frac{123}{76}\right) = (75\pi)(1.5)$	<p>M1</p> <p>A2</p>	<p>CM of composite body</p> <p>Give A1 if just one error</p>	FT values from (i)
	OR	$\int_0^3 \pi x(25 - (x^2 + 16)) dx$ $= \frac{81}{4}\pi$ $(18\pi)\bar{x}_B = \frac{81}{4}\pi$	<p>M1</p> <p>A1</p> <p>A1 FT</p>		
		$\bar{x}_B = \frac{9}{8} \quad (= 1.125)$	<p>A1</p> <p>[4]</p>	CAO	